Post-Quantum Signatures from Secure Multiparty Computation

Thibauld Feneuil

Journées C2 - 2023

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CryptoExperts
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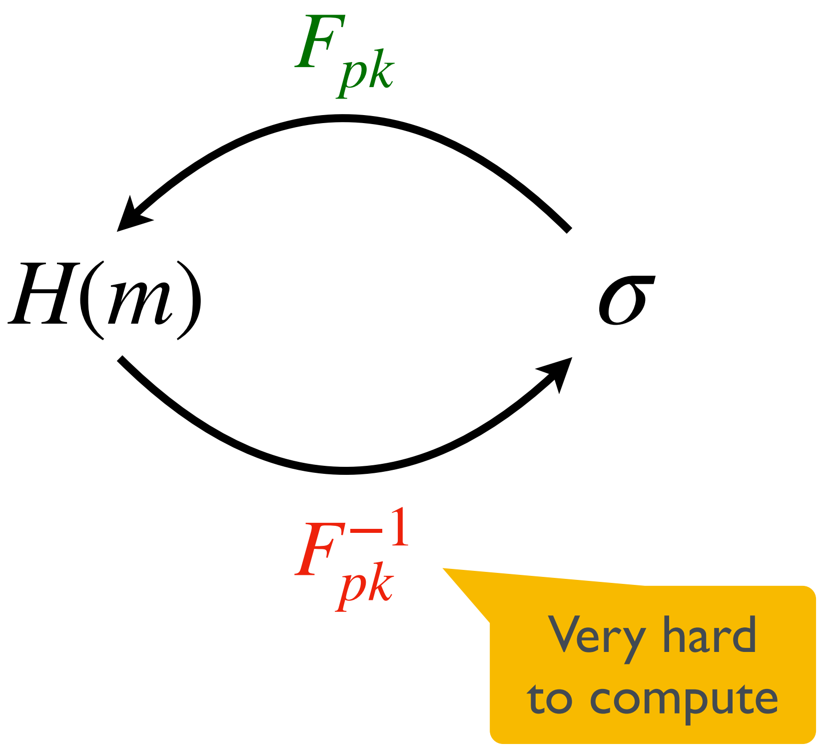
Introduction

How to build signature schemes?

Hash & Sign



- Short signatures
- "Trapdoor" in the public key

How to build signature schemes?

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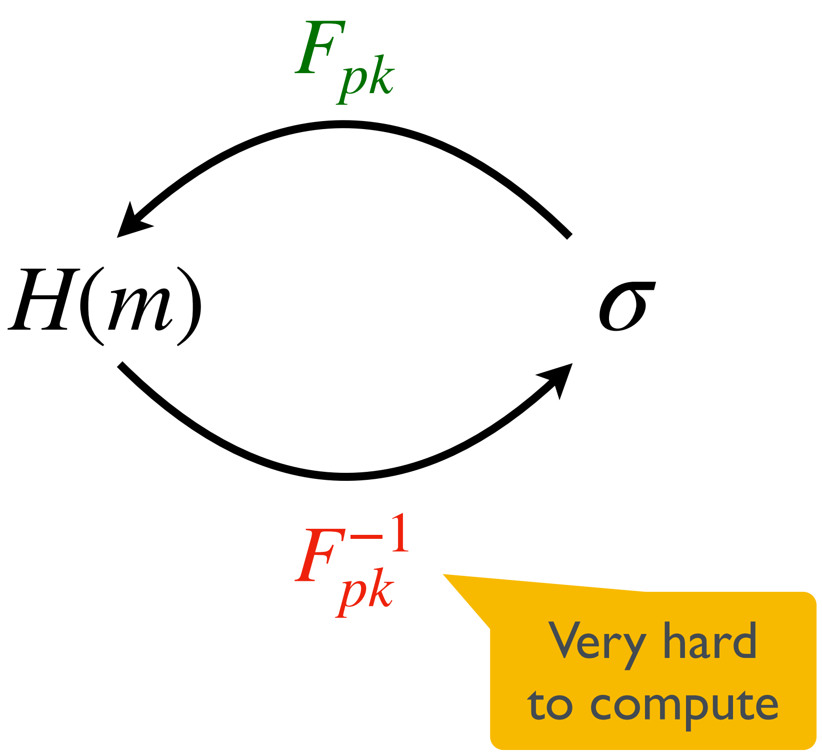
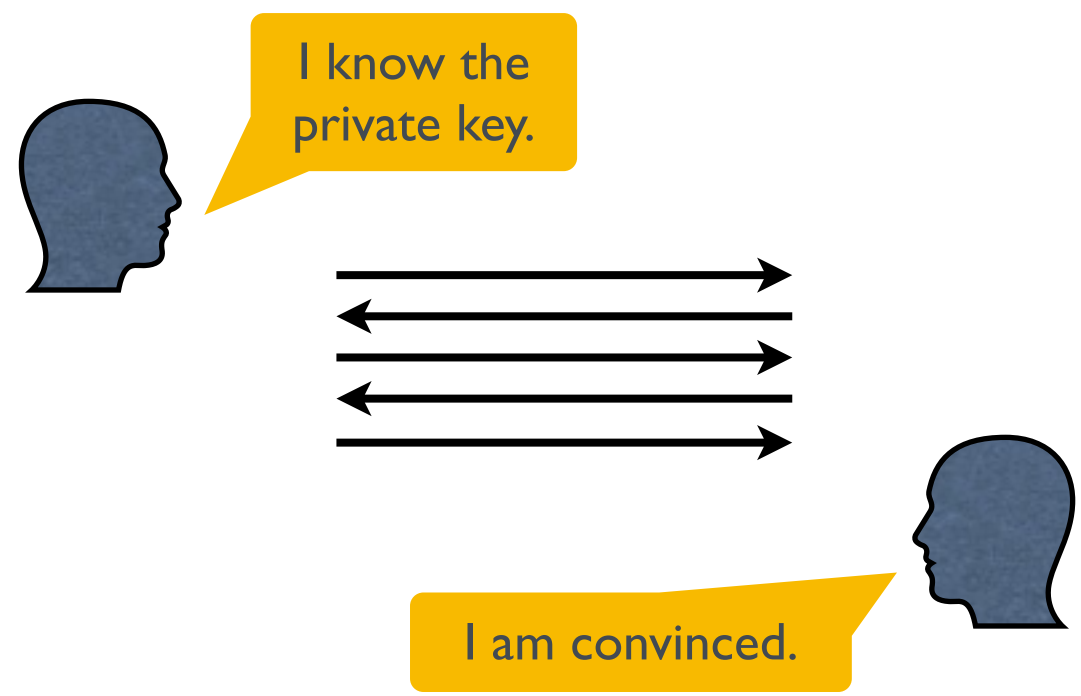


- Short signatures
- “Trapdoor” in the public key

From a zero-knowledge proof



- Large(r) signatures
- Short public key

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Hash & Sign

- Short signatures
- "Trapdoor" in the public key

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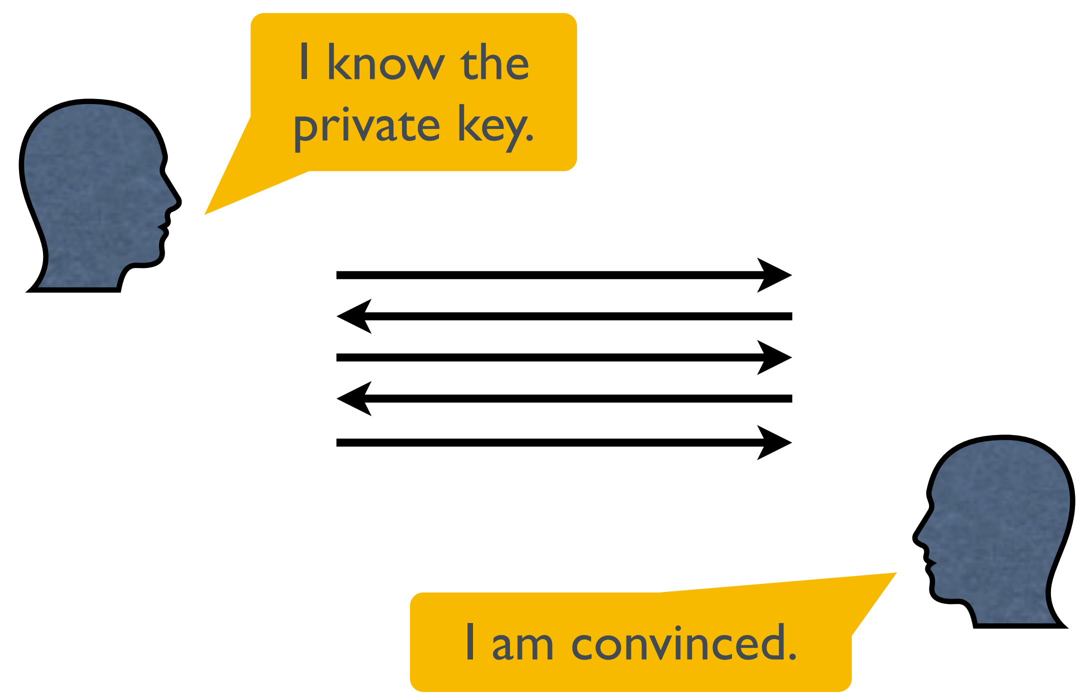

- Large(r) signatures
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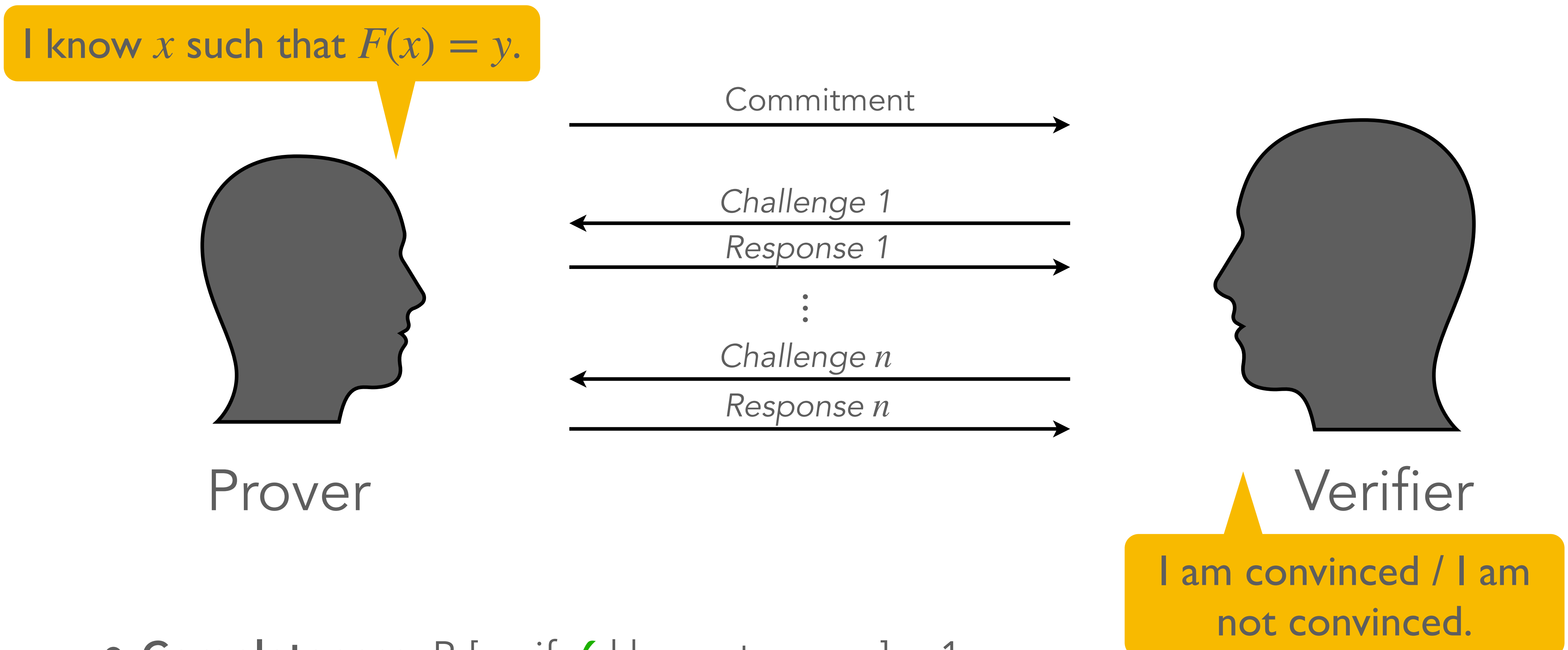
Proof of knowledge

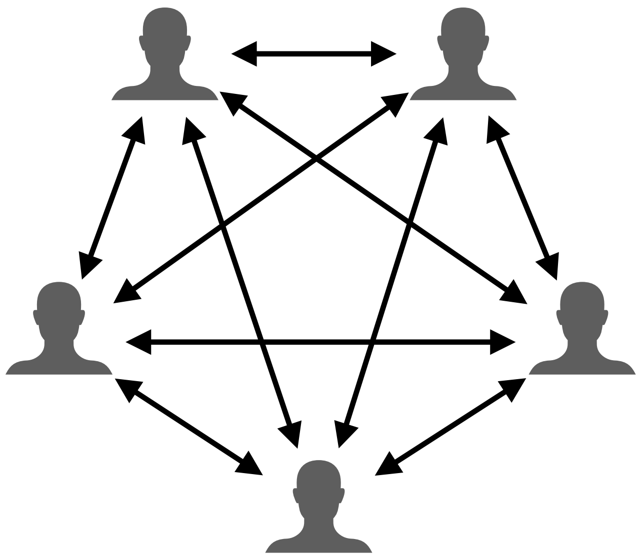


- **Completeness:** Pr[verif ✓ | honest prover] = 1
- **Soundness:** Pr[verif ✓ | malicious prover] $\leq \varepsilon$ (e.g. 2^{-128})
- **Zero-knowledge:** verifier learns nothing on x

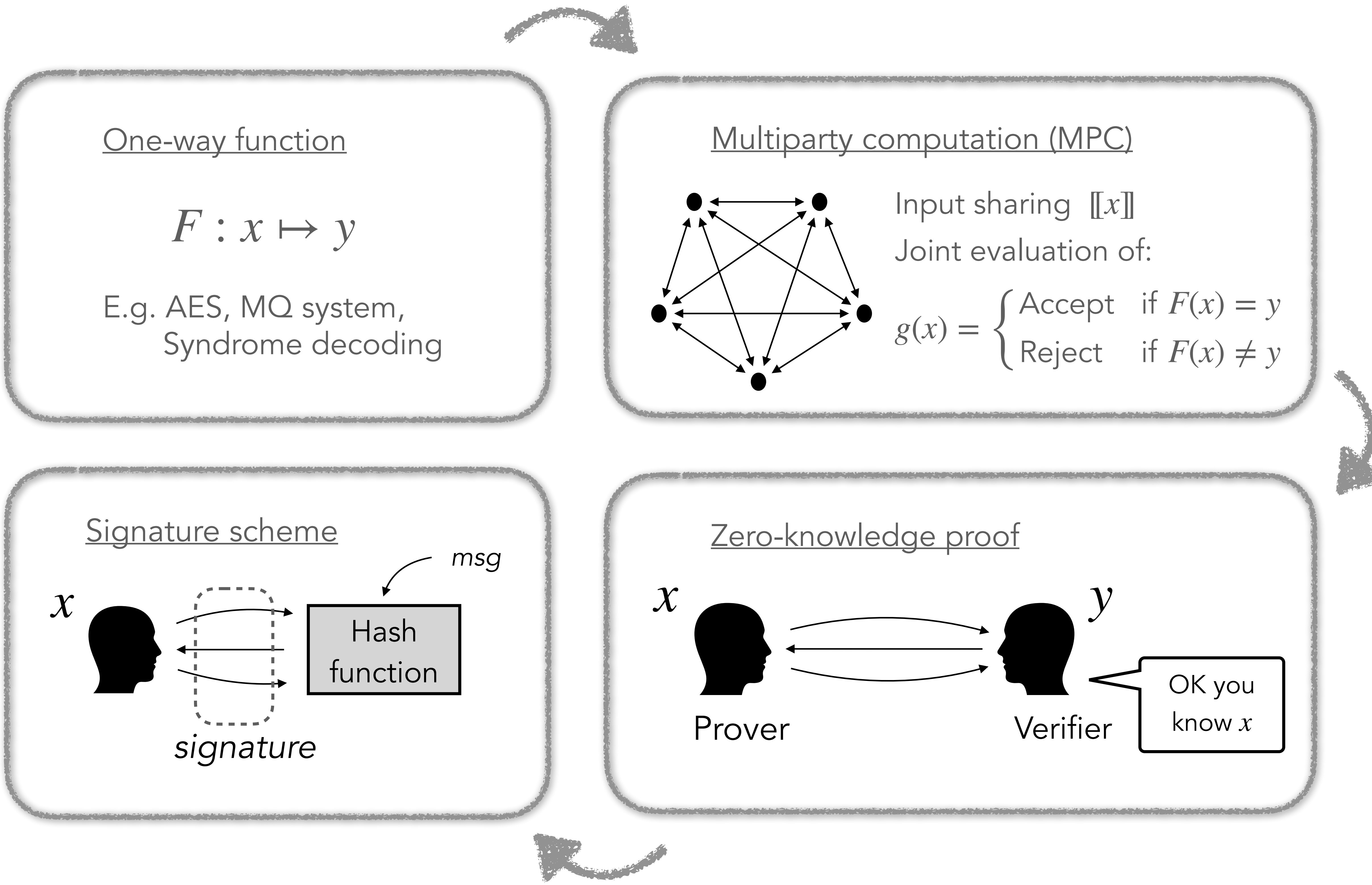
MPC in the Head

- **[IKOS07]** Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, Amit Sahai: "Zero-knowledge from secure multiparty computation" (STOC 2007)

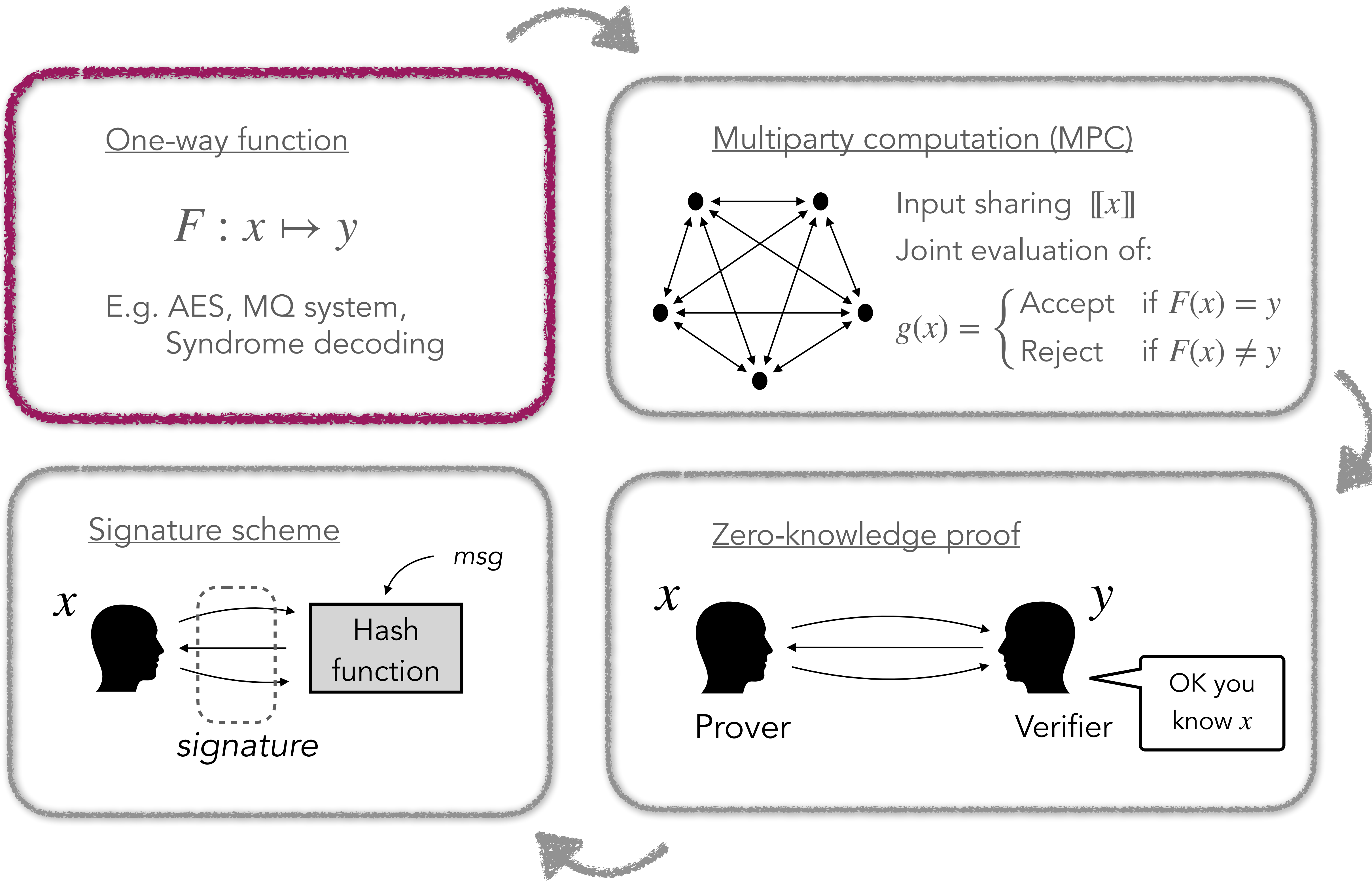
- Turn a *multiparty computation* (MPC) into an identification scheme / zero knowledge proof of knowledge



- **Generic**: can be apply to any cryptographic problem



One-way function
$F : x \mapsto y$
E.g. AES, MQ system, Syndrome decoding
Multiparty computation (MPC)
Input sharing $[\![x]\!]$
Joint evaluation of:
$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$
Zero-knowledge proof
x
y
Prover
Verifier
OK you know x
Signature scheme
x
msg
Hash function
signature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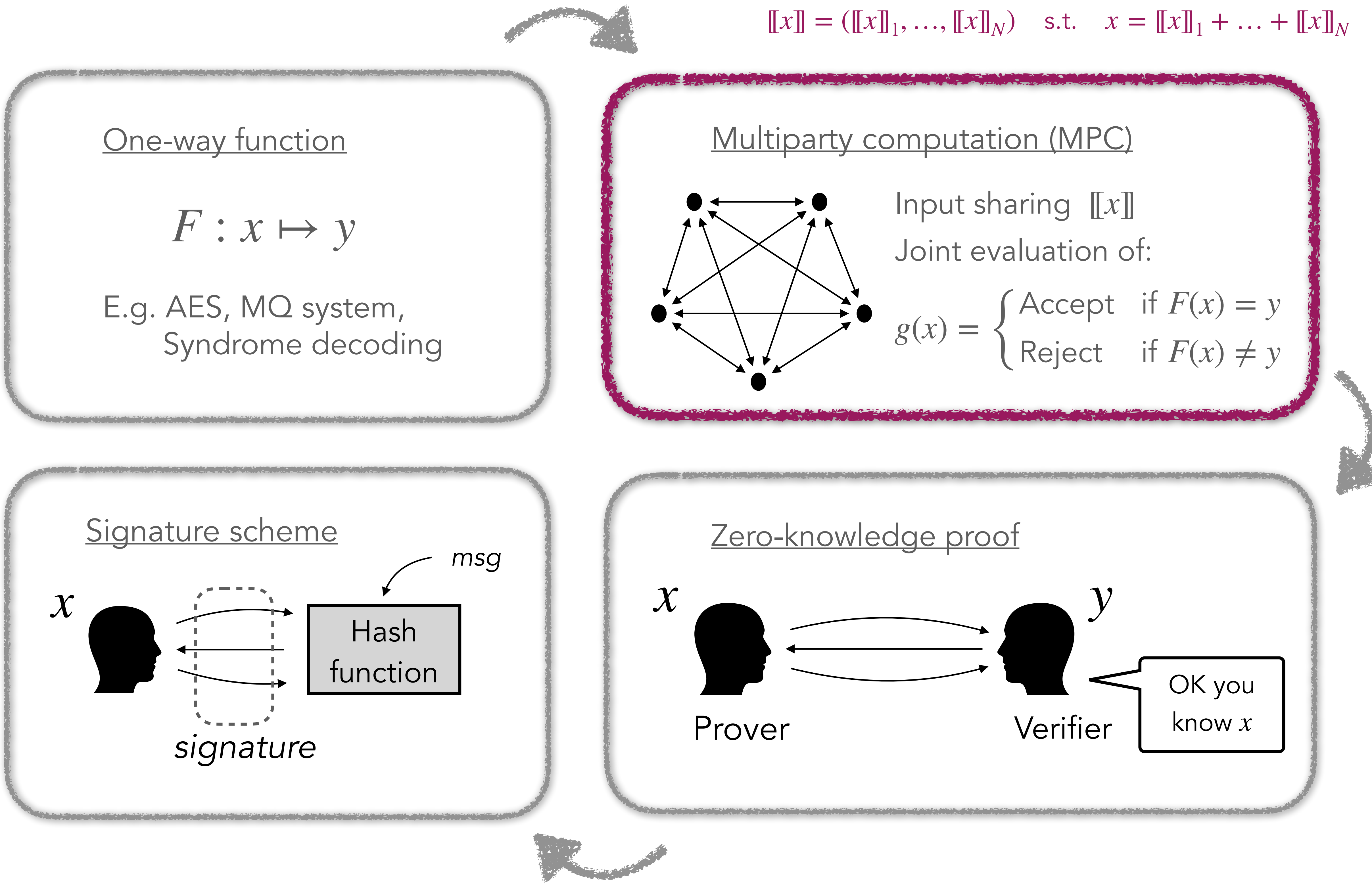

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$ s.t. $x = [\![x]\!]_1 + \ldots + [\![x]\!]_N$
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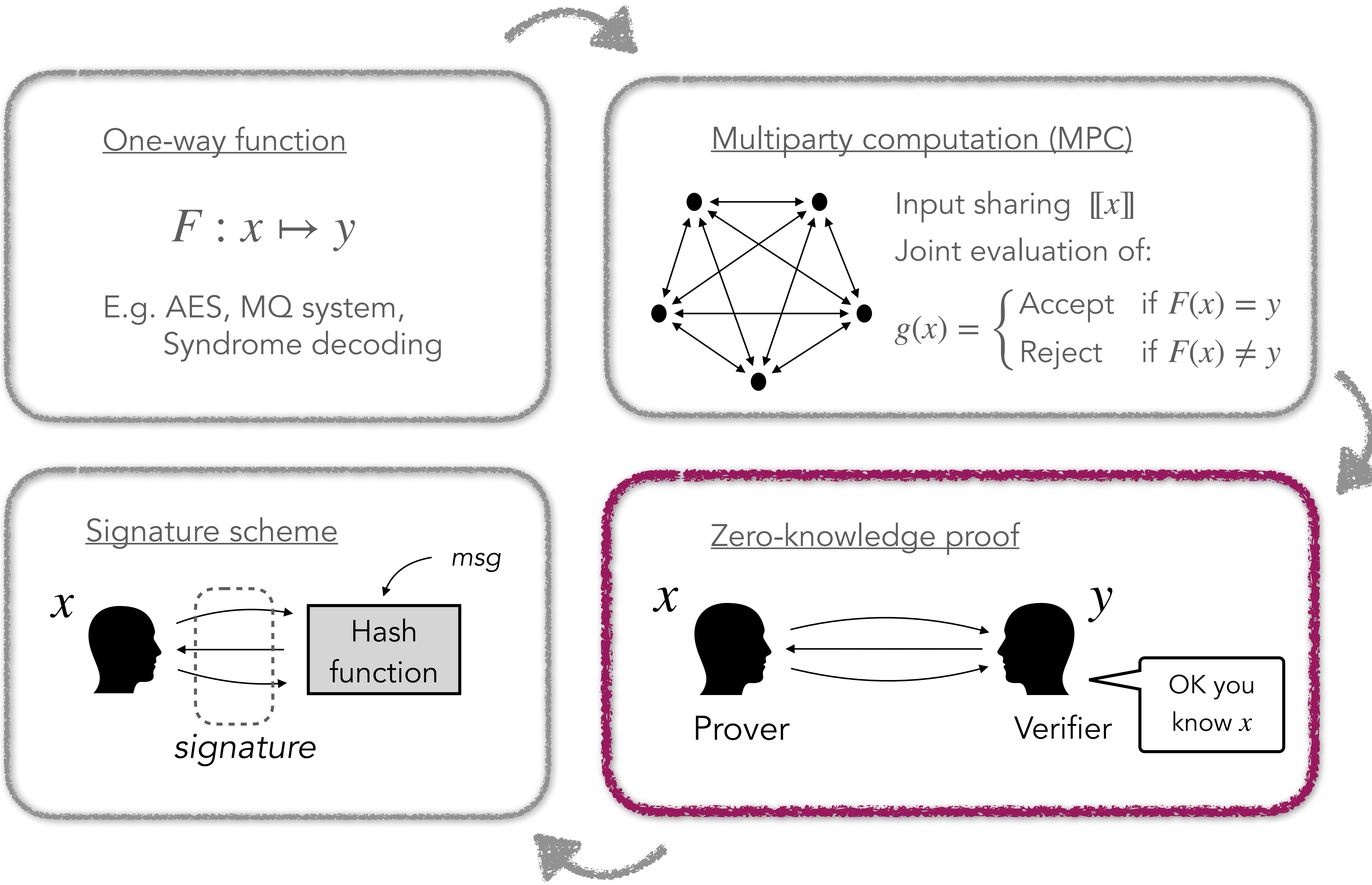

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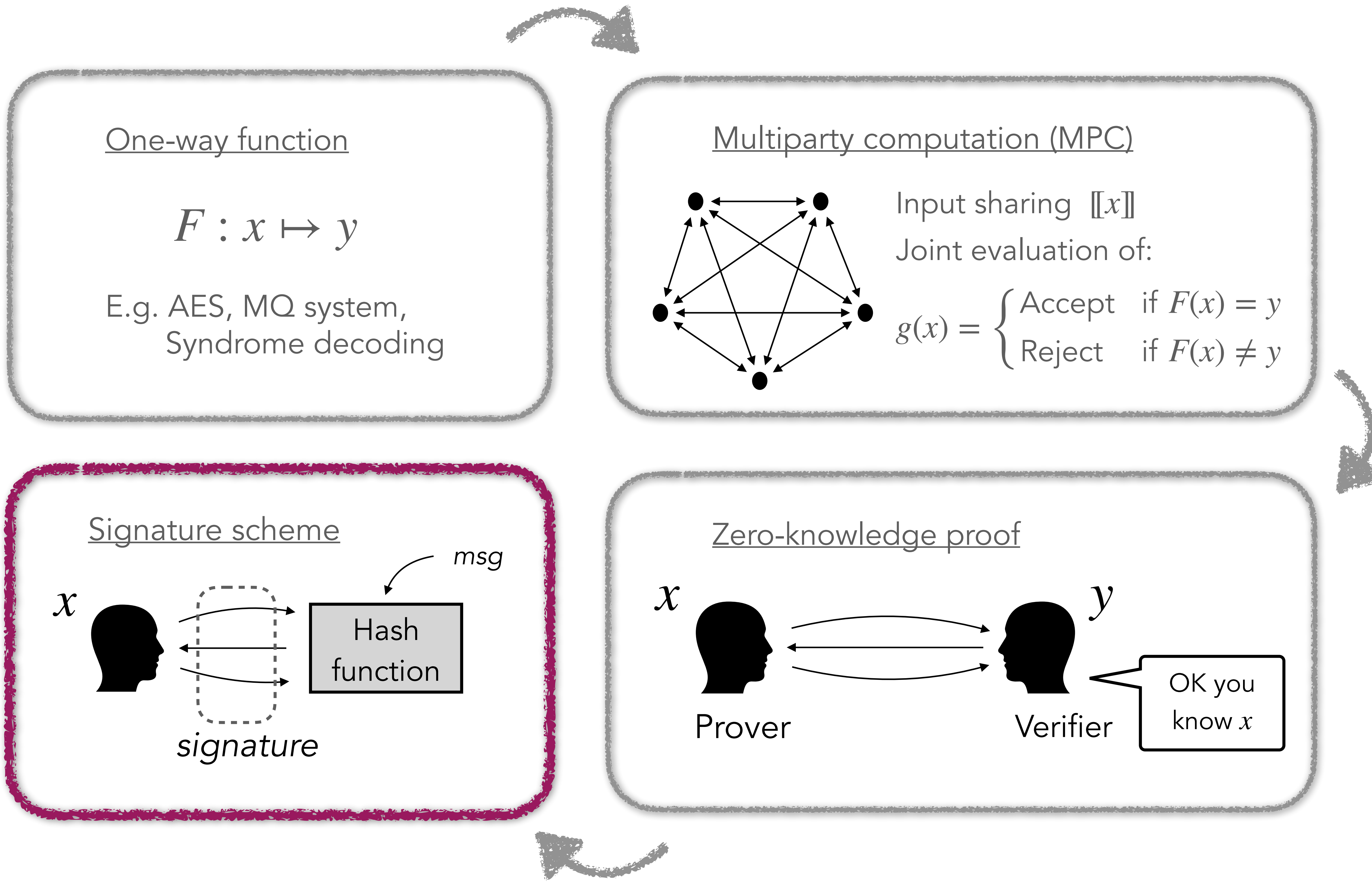

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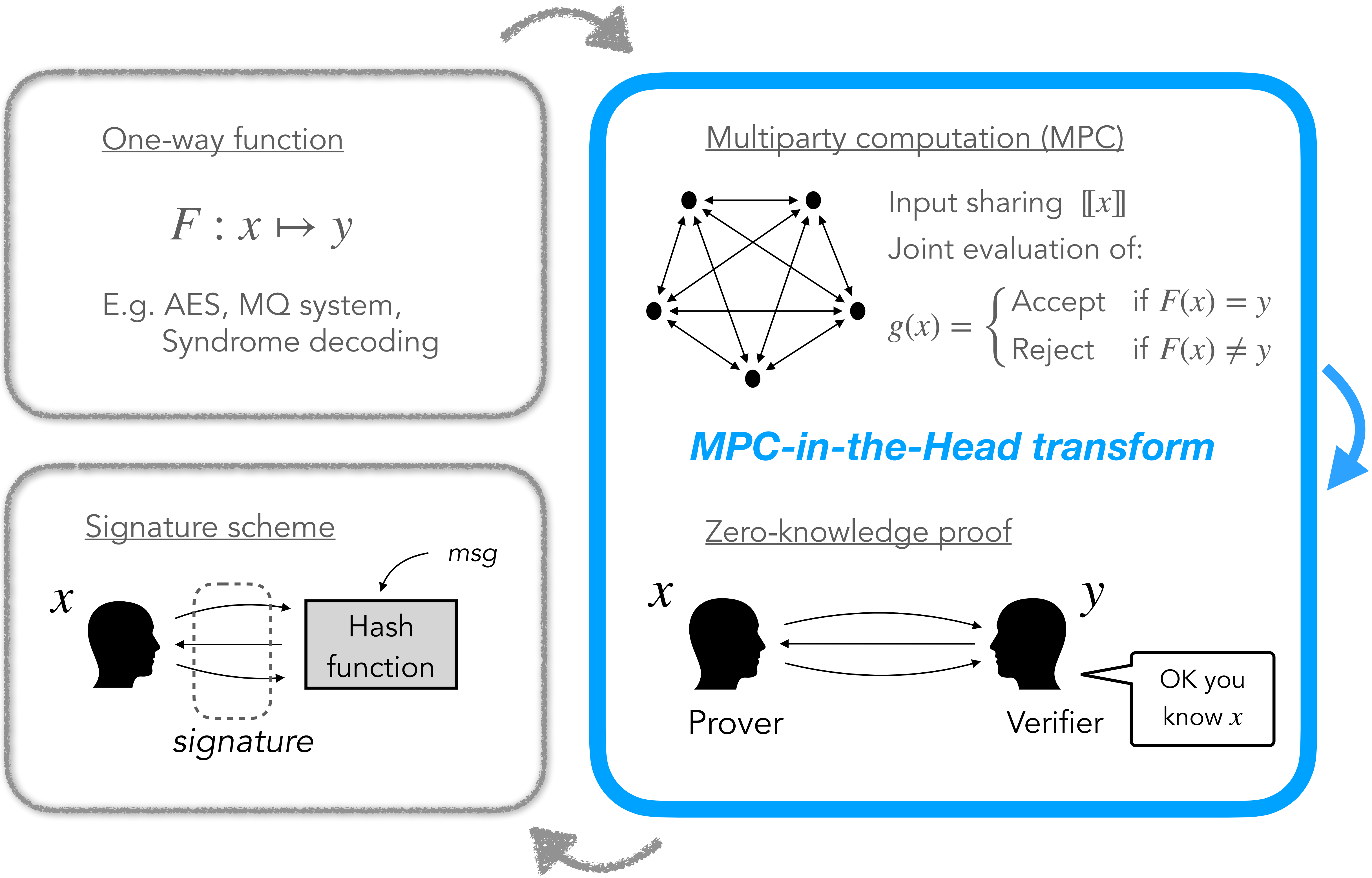

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MPC-in-the-Head transform
Zero-knowledge proof
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y
Prover
Verifier
OK you
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MPCitH transform

Prover

Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\cdots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

Prover

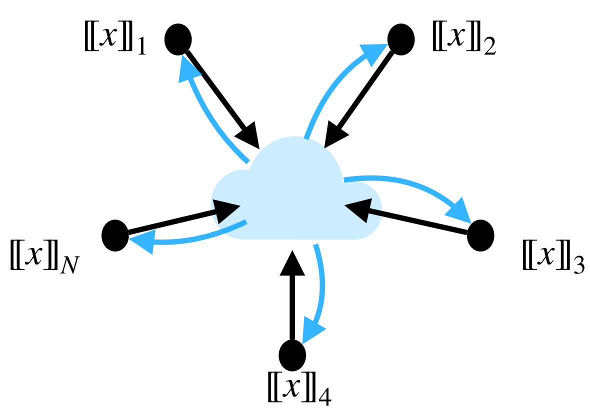
Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

② Run MPC in their head



$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

...

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

Prover

Verifier

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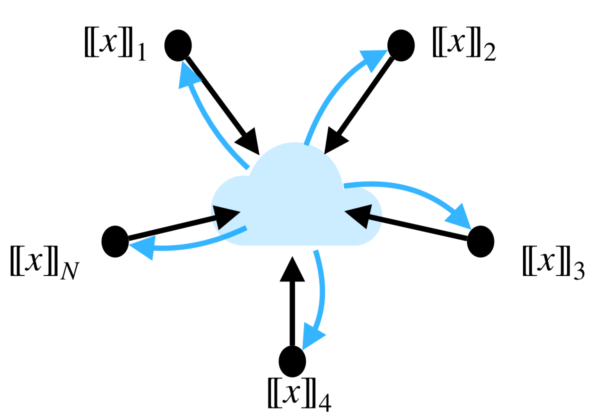

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$
…
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

send broadcast
$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

i^*

③ Choose a random party

$i^* \xleftarrow{\$} \{1,\ldots,N\}$

Prover

Verifier

MPCitH transform

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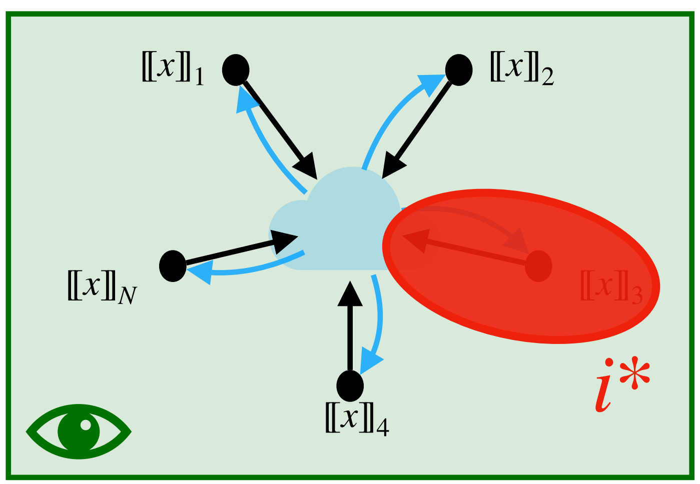

④ Open parties $\{1,\ldots,N\} \backslash \{i^*\}$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\cdots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$ →

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$ →

← i^*

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$ →

③ Choose a random party

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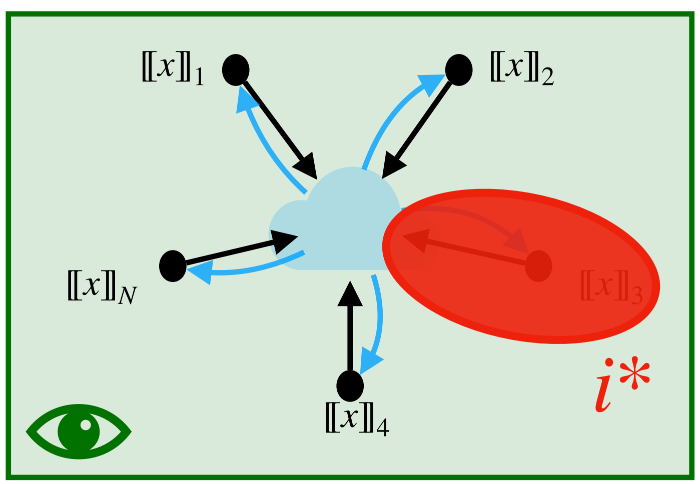

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③ Choose a random party

$i^* \leftarrow^{\$} \{1,\ldots,N\}$

⑤ Check $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $g(y, \alpha) =$ Accept

Prover

Verifier

MPCitH transform

① Generate and commit shares

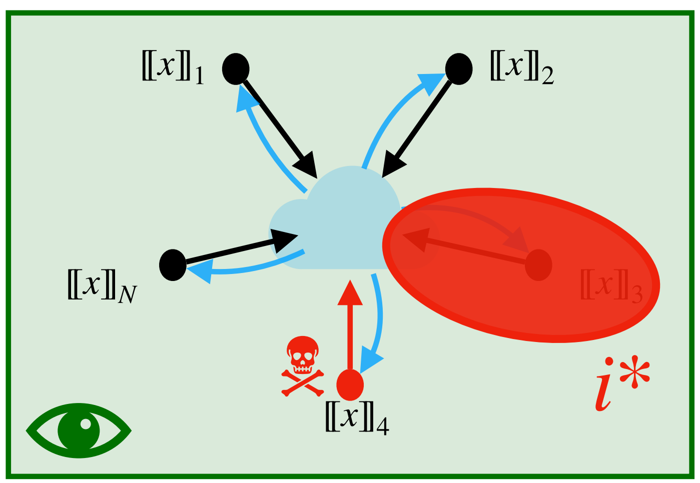
$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

We have $F(x) \neq y$ where

$x := [\![x]\!]_1 + \ldots + [\![x]\!]_N$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$
…
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$ →

② Run MPC in their head



send broadcast
$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$ →

③ Choose a random party

$i^* \xleftarrow{\$} \{1, \ldots, N\}$

← i^*

④ Open parties $\{1, \ldots, N\} \backslash \{i^*\}$

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$ →

⑤ Check $\forall i \neq i^*$

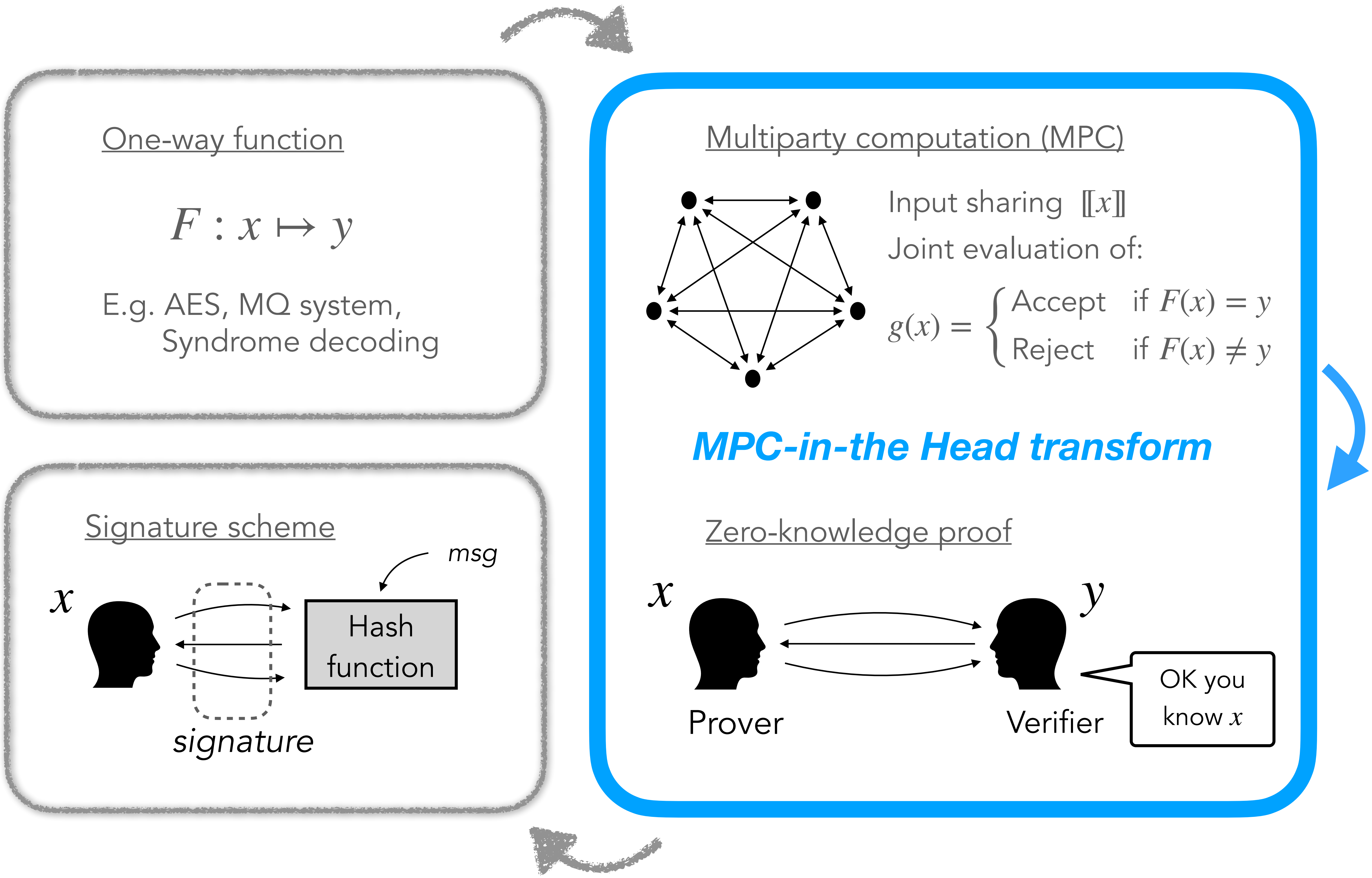
- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $g(y, \alpha) =$ Accept

Malicious Prover

Verifier

Cheating detected!



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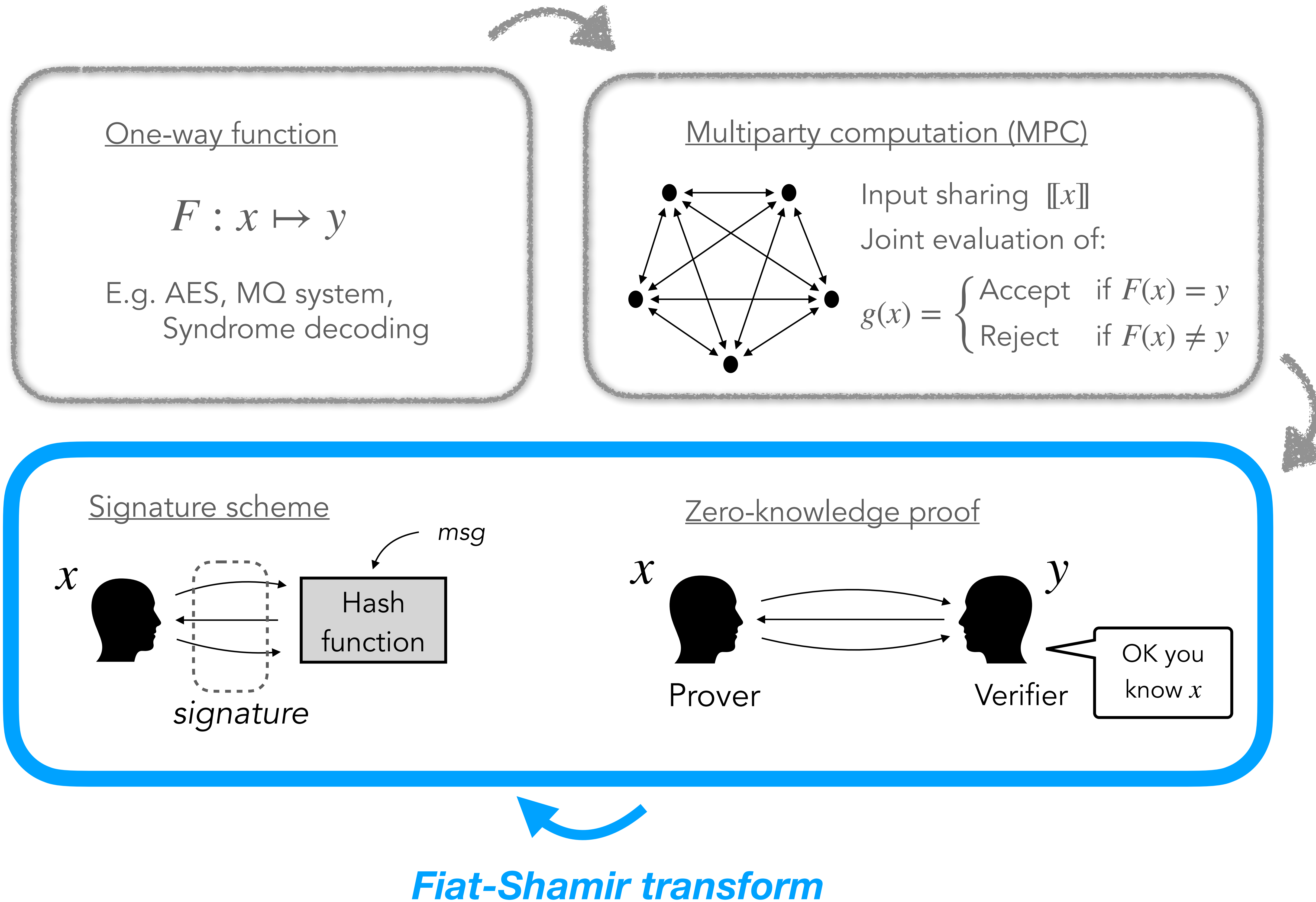
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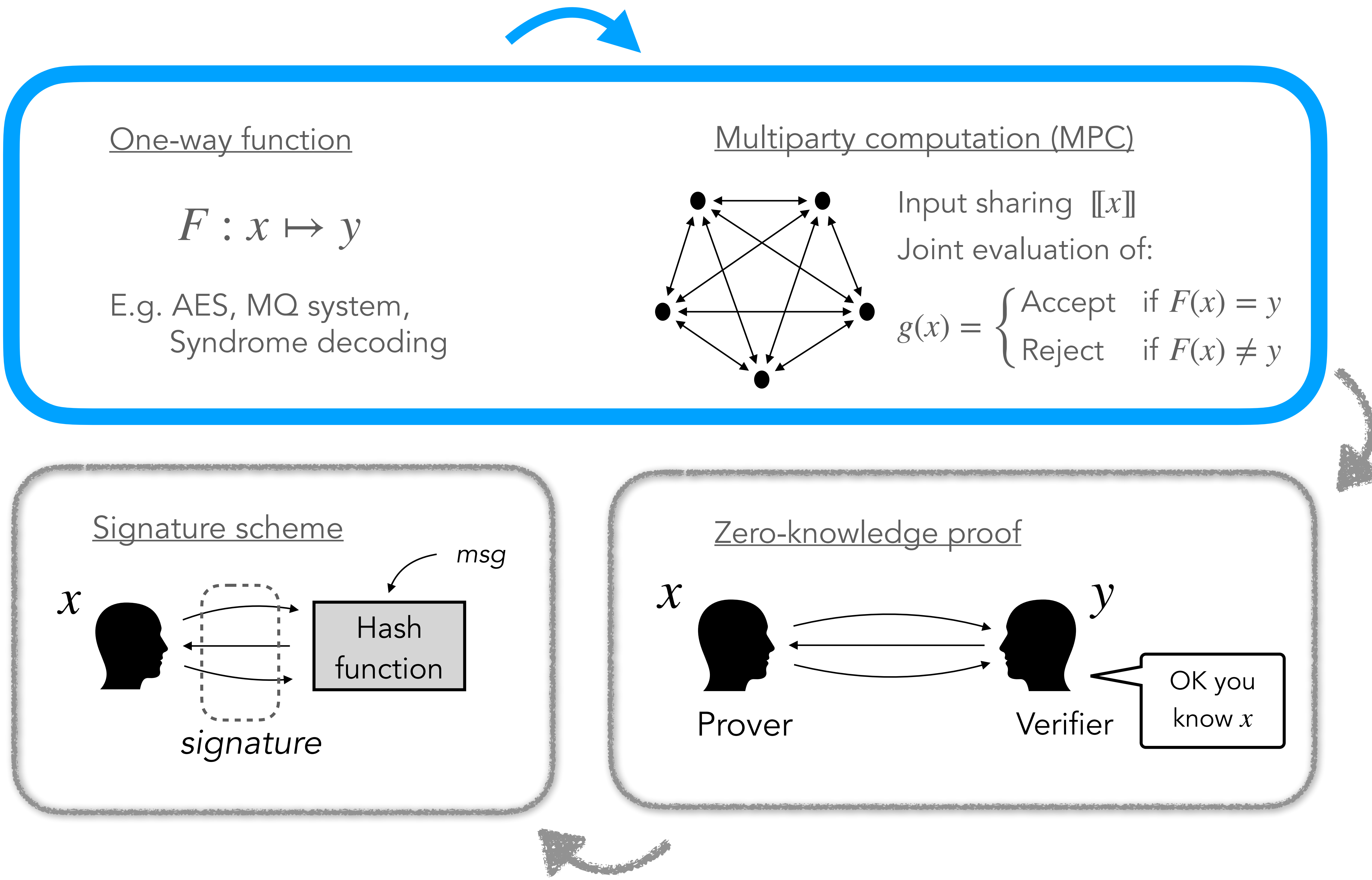
Fiat-Shamir transform



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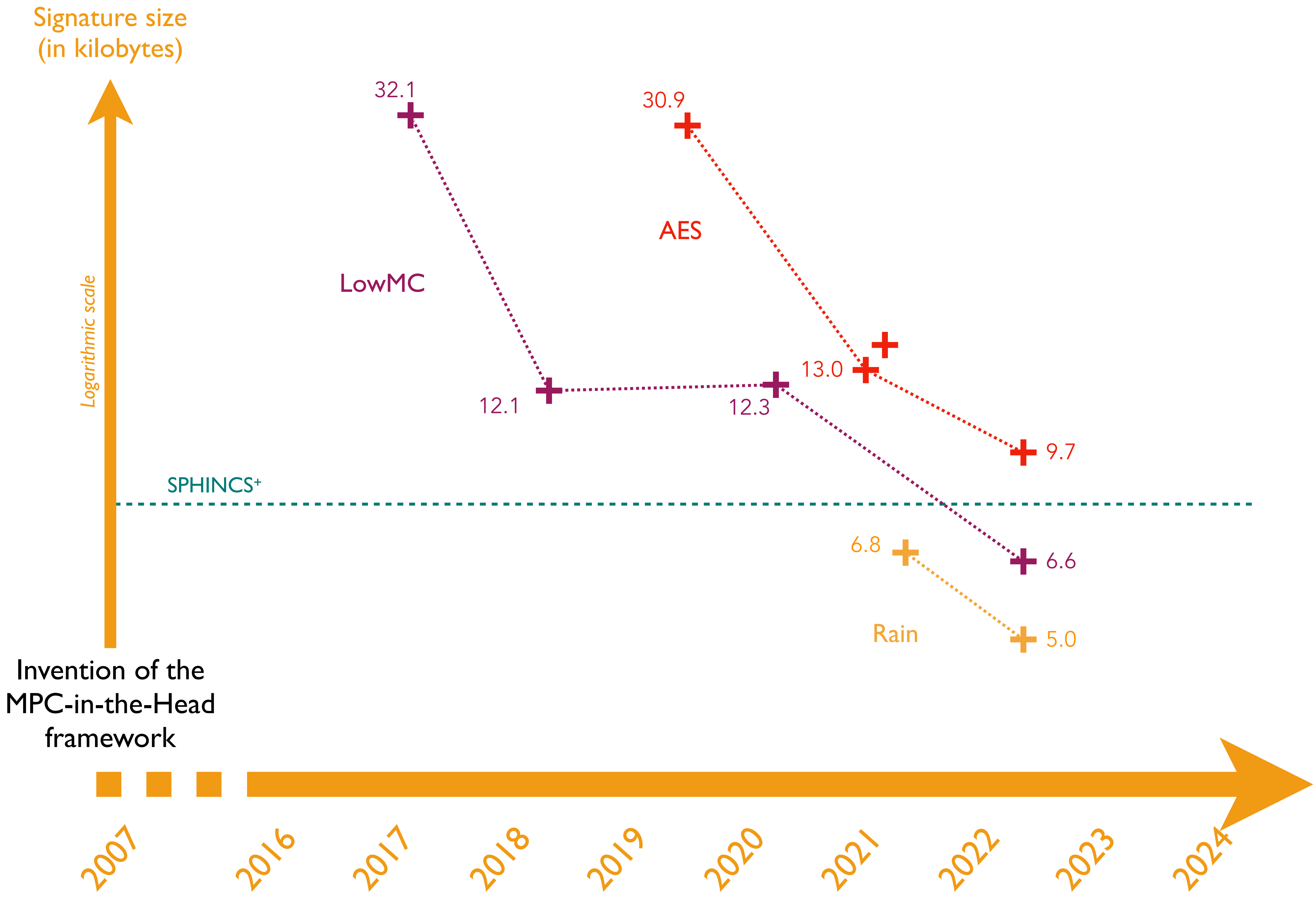
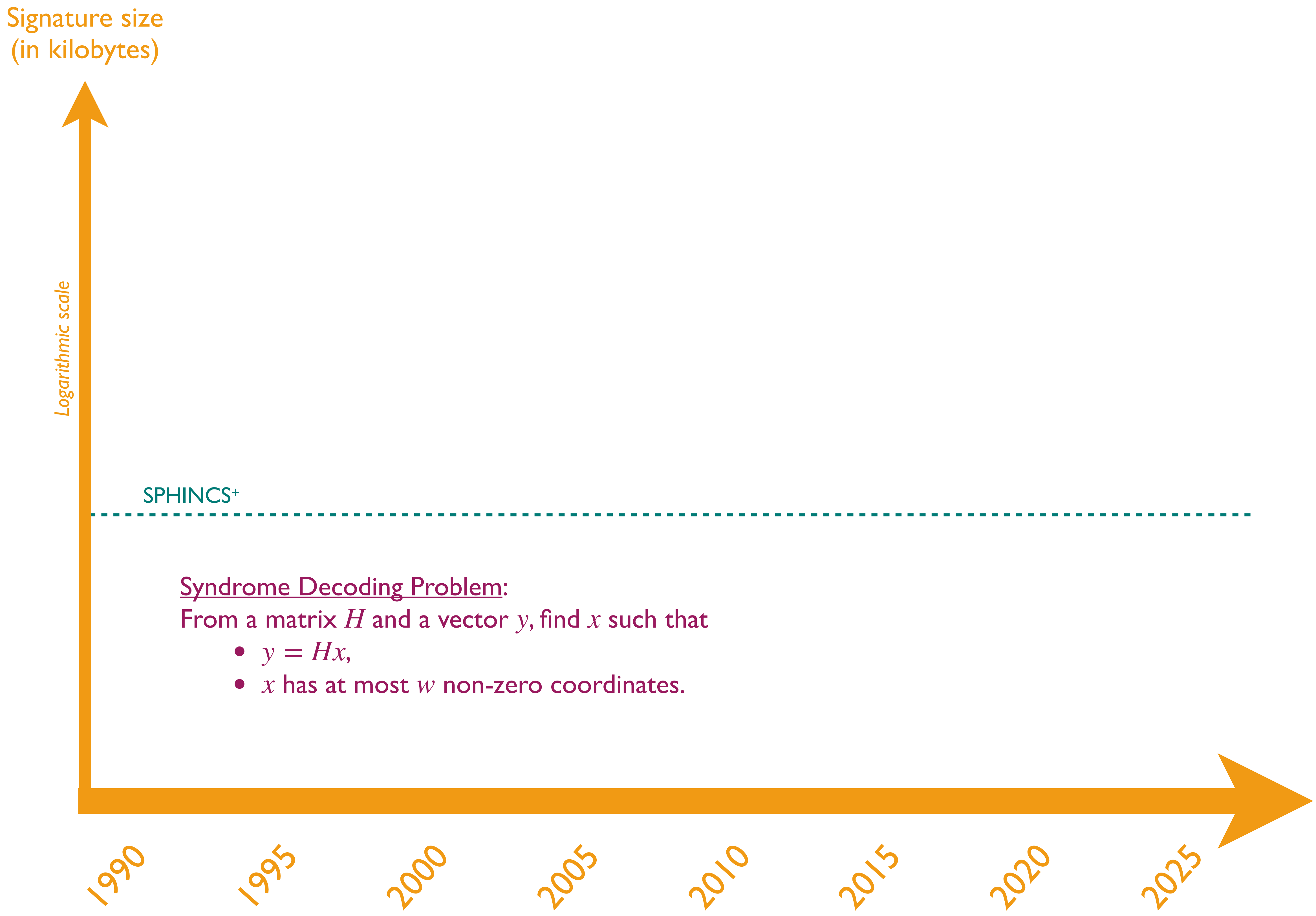
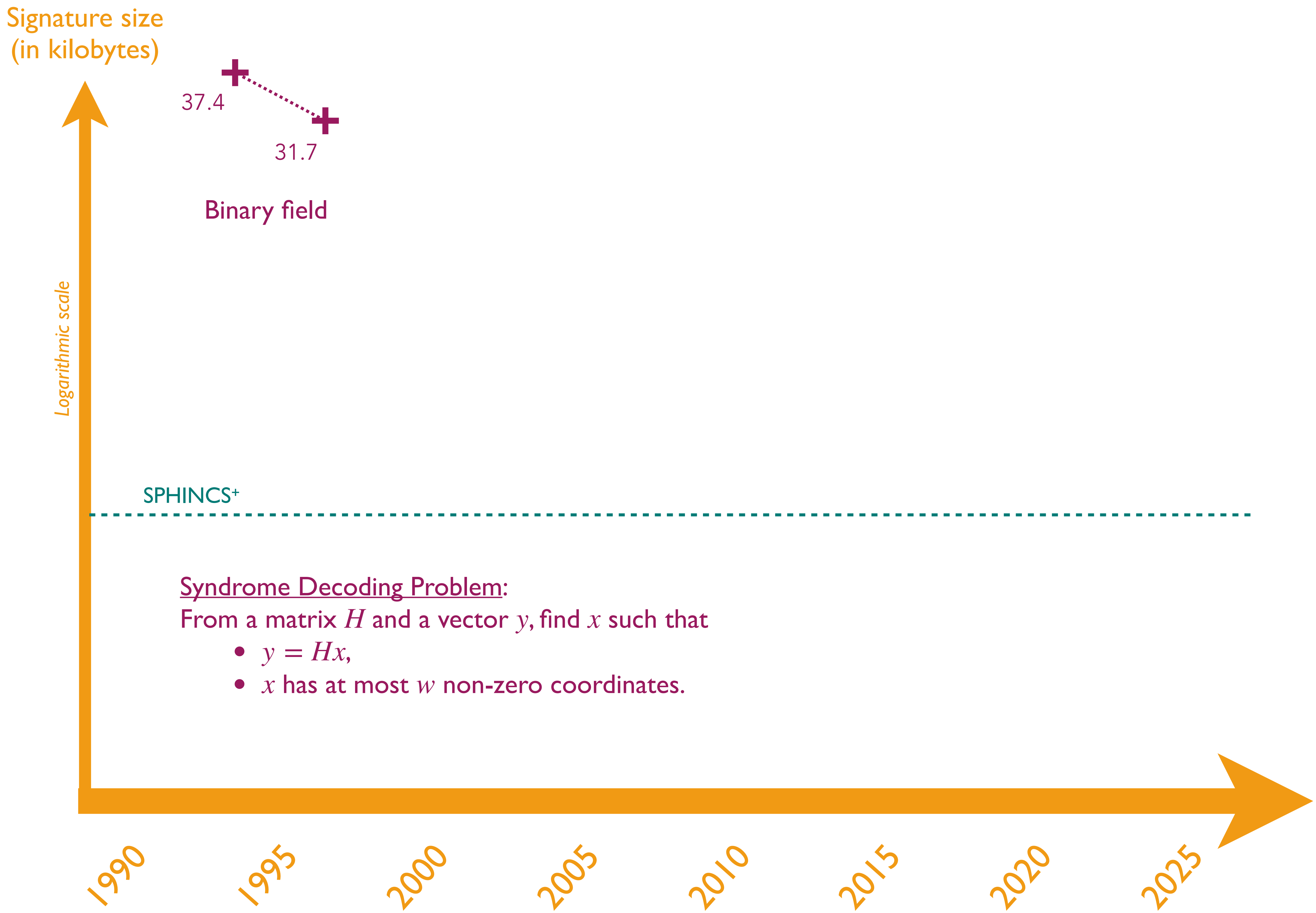

Signature size
(in kilobytes)
Logarithmic scale
32.1
30.9
AES
LowMC
13.0
12.1
12.3
9.7
SPHINCS+
6.8
6.6
Rain
5.0
Invention of the
MPC-in-the-Head
framework
2007
2016
2017
2018
2019
2020
2021
2022
2023
2024



Syndrome Decoding Problem:
From a matrix H and a vector y, find x such that

- $y = Hx$,
- x has at most w non-zero coordinates.



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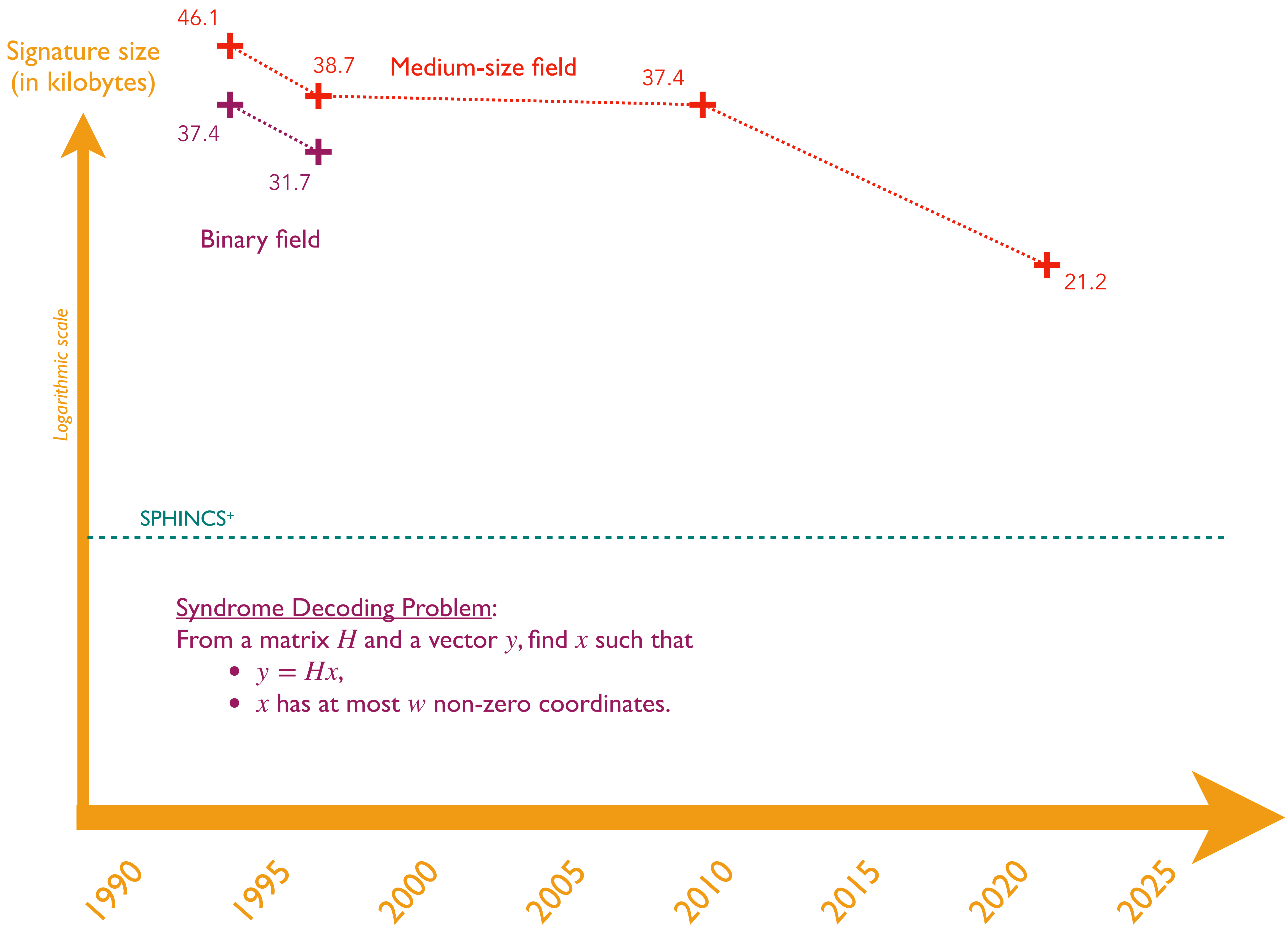

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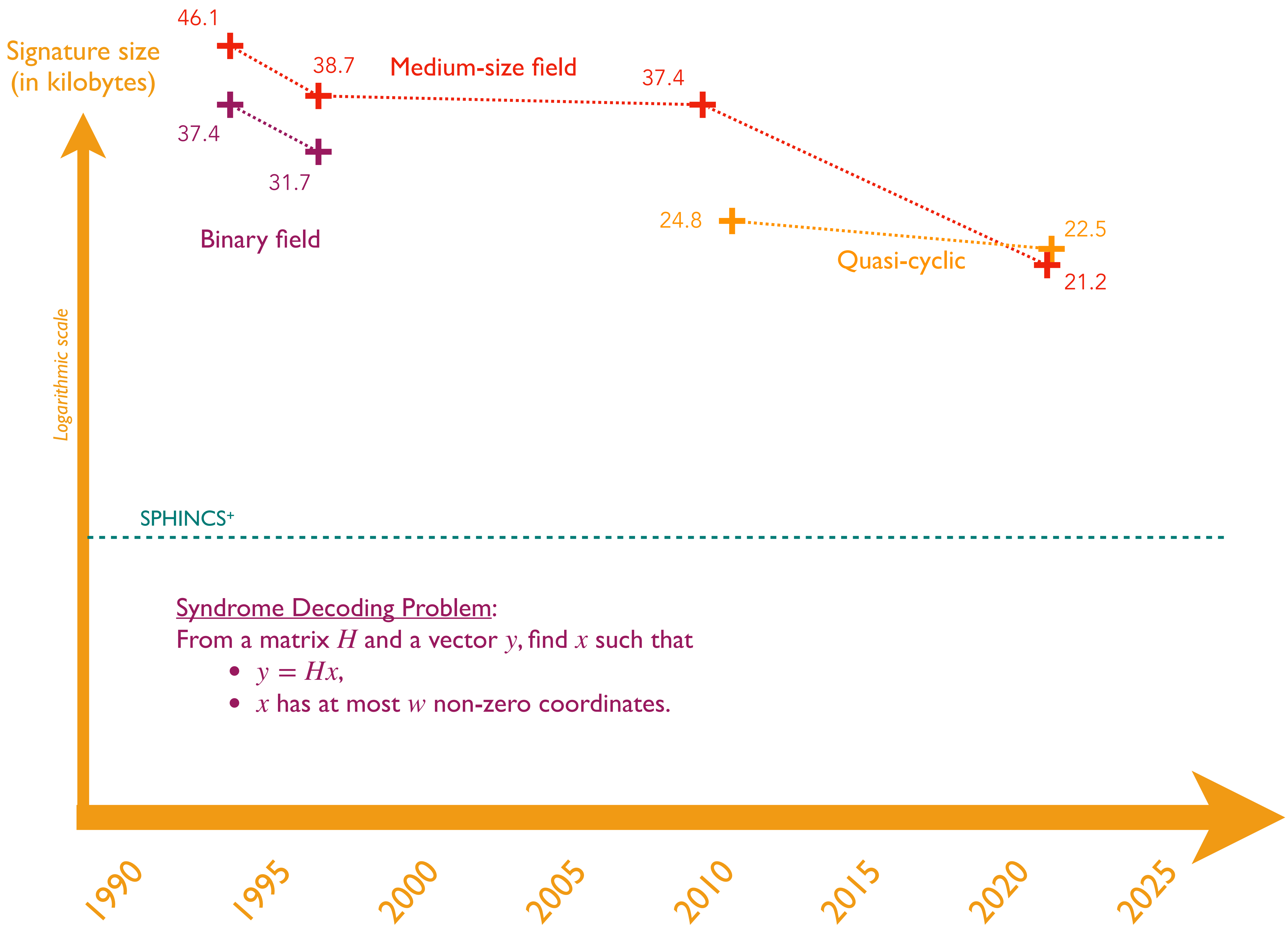

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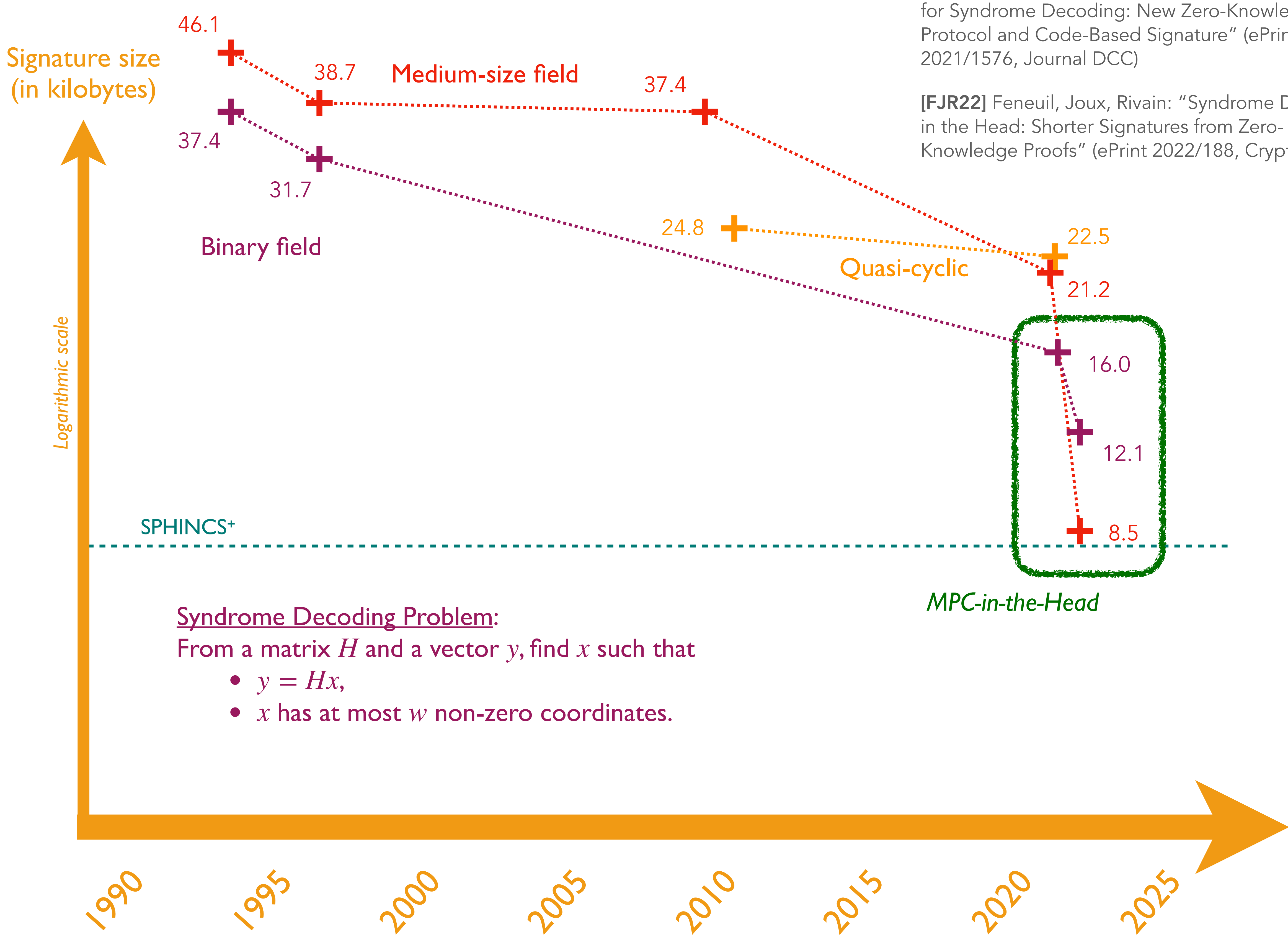

[FJR23] Feneuil, Joux, Rivain: "Shared Permutation for Syndrome Decoding: New Zero-Knowledge Protocol and Code-Based Signature" (ePrint 2021/1576, Journal DCC)

[FJR22] Feneuil, Joux, Rivain: "Syndrome Decoding in the Head: Shorter Signatures from Zero-Knowledge Proofs" (ePrint 2022/188, Crypto 2022)

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Exploring other assumptions

- **Subset Sum Problem:** ≥ 100 KB $\Rightarrow$ 19.1 KB
- **Multivariate Quadratic Problem:** 6.3 – 7.3 KB
- **MinRank Problem:** $\approx 5 - 6$ KB
- **Rank Syndrome Decoding Problem:** $\approx 5 - 6$ KB
- **Permuted Kernel Problem (or variant):** ≈ 6 KB
- …

MPCitH-based NIST Candidates

1st June 2023:

Deadline for the NIST call
for additional post-quantum signatures

MPCitH-based NIST Candidates

	Assumption	Size (in KB)
AIMer	AIM (MPC-friendly one-way function)	4.2
Biscuit	Structured MQ problem (PowAff2)	4.7
MIRA	MinRank problem	5.6
MiRitH	MinRank problem	5.7
RYDE	Syndrome decoding problem in rank metric	6.0
PERK*	Permuted Kernel problem (variant)	6.1
MQOM	Unstructured MQ problem	6.3
SDitH	Syndrome decoding problem in Hamming	8.2

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PERK	Permuted Kernel problem (variant)	6.1
MQOM	Unstructured MQ problem	6.3
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What about the computational cost ?

Traditional Transformation

(2018) Emulation : N parties

Traditional Transformation (2018)

Emulation : N parties

Hypercube Technique (2022)

$1 + \log_2 N$ parties

No communication penalty

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (ePrint 2022/1645, Eurocrypt 2023)

Traditional Transformation

(2018)

Emulation : N parties

Hypercube Technique

(2022)

$1 + \log_2 N$ parties

No communication penalty

Threshold Approach

(2022)

Shamir's secret sharings

$1 + \ell$ parties

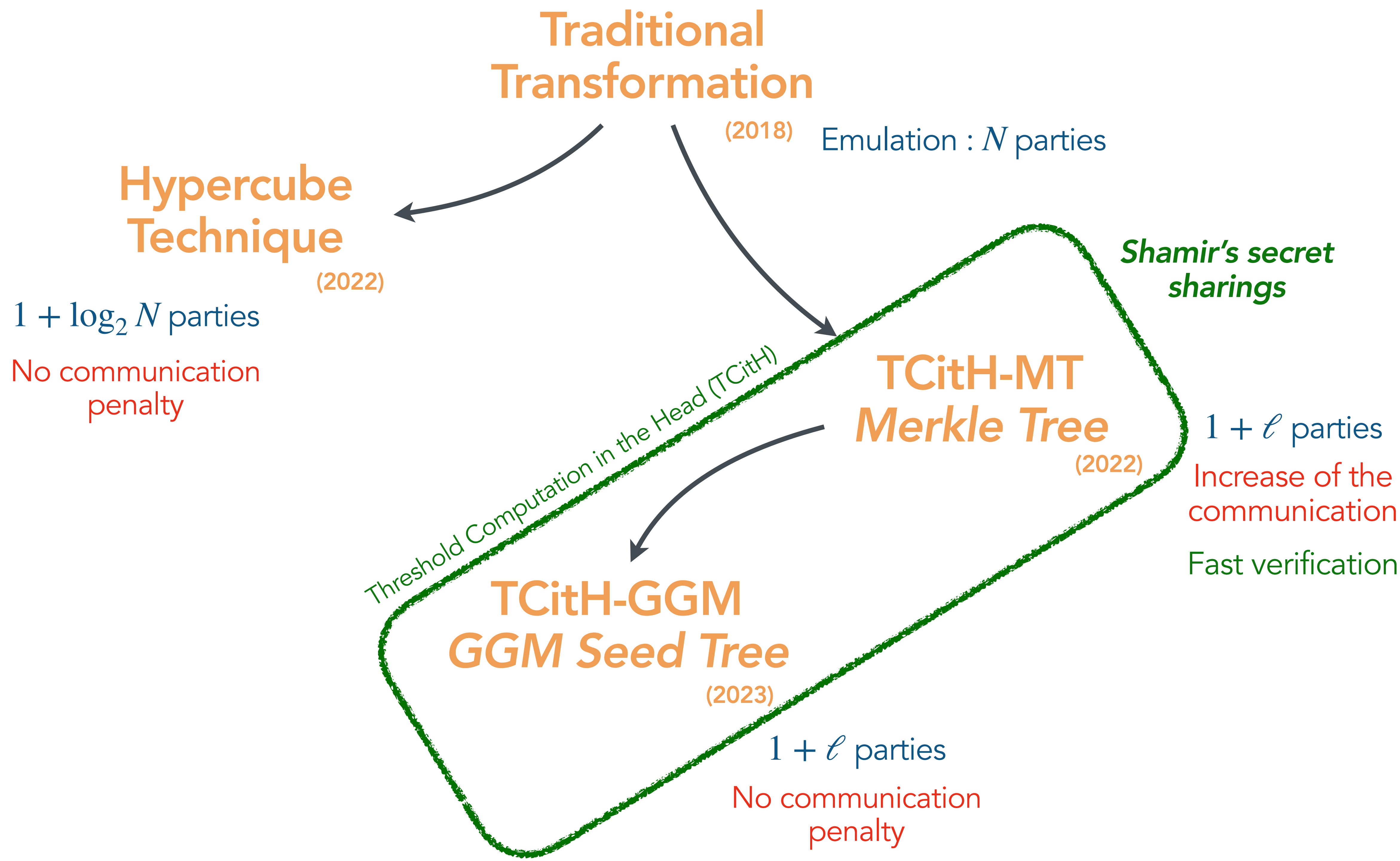
Increase of the communication

Fast verification

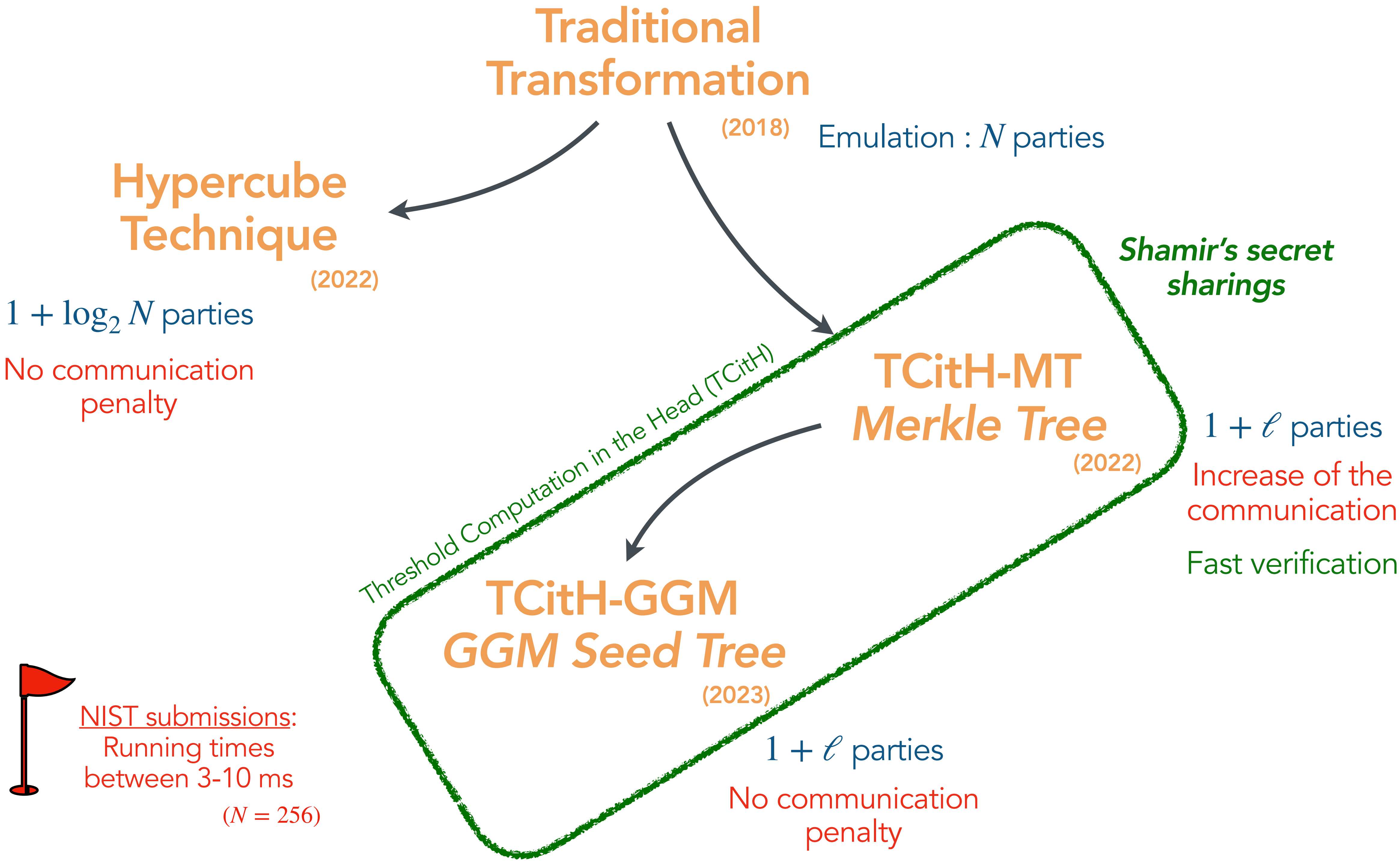
Shamir's secret sharing: to share a value s,

- Build a random degree-ℓ polynomial $P(X) := s + \sum_{j=1}^{\ell} r_j X^j$.
- Set the i^{th} share $[\![s]\!]_i$ as $[\![s]\!]_i := P(e_i)$, where $e_i \neq 0$.

[FR22] Feneuil, Rivain: "Threshold Linear Secret Sharing to the Rescue of MPC-in-the-Head" (ePrint 2022/1407, Asiacrypt 2023)



[FR23] Feneuil, Rivain: "Threshold Computation in the Head: Improved Framework for Post-Quantum Signatures and Zero-Knowledge Arguments" (ePrint 2023/1573)



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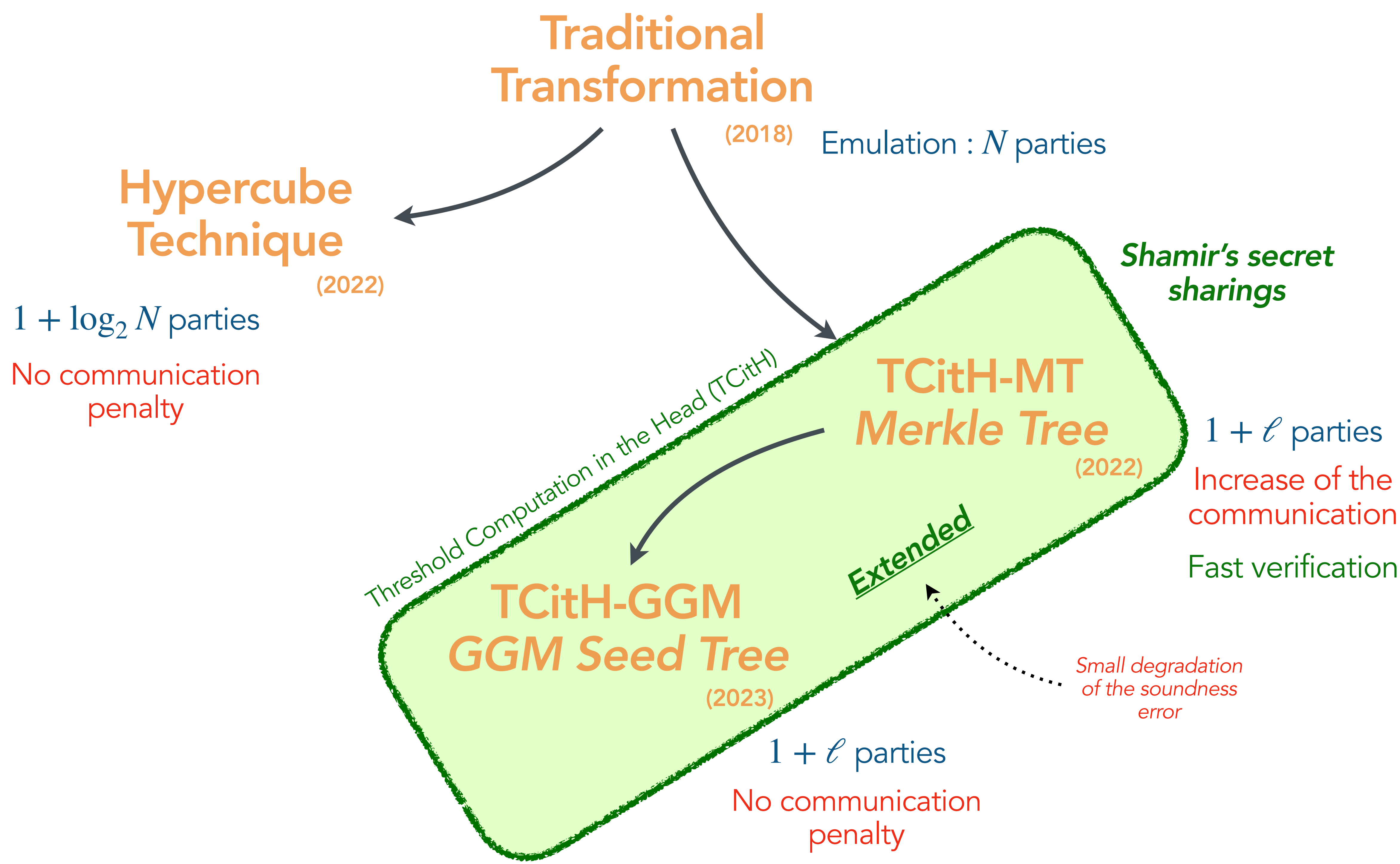

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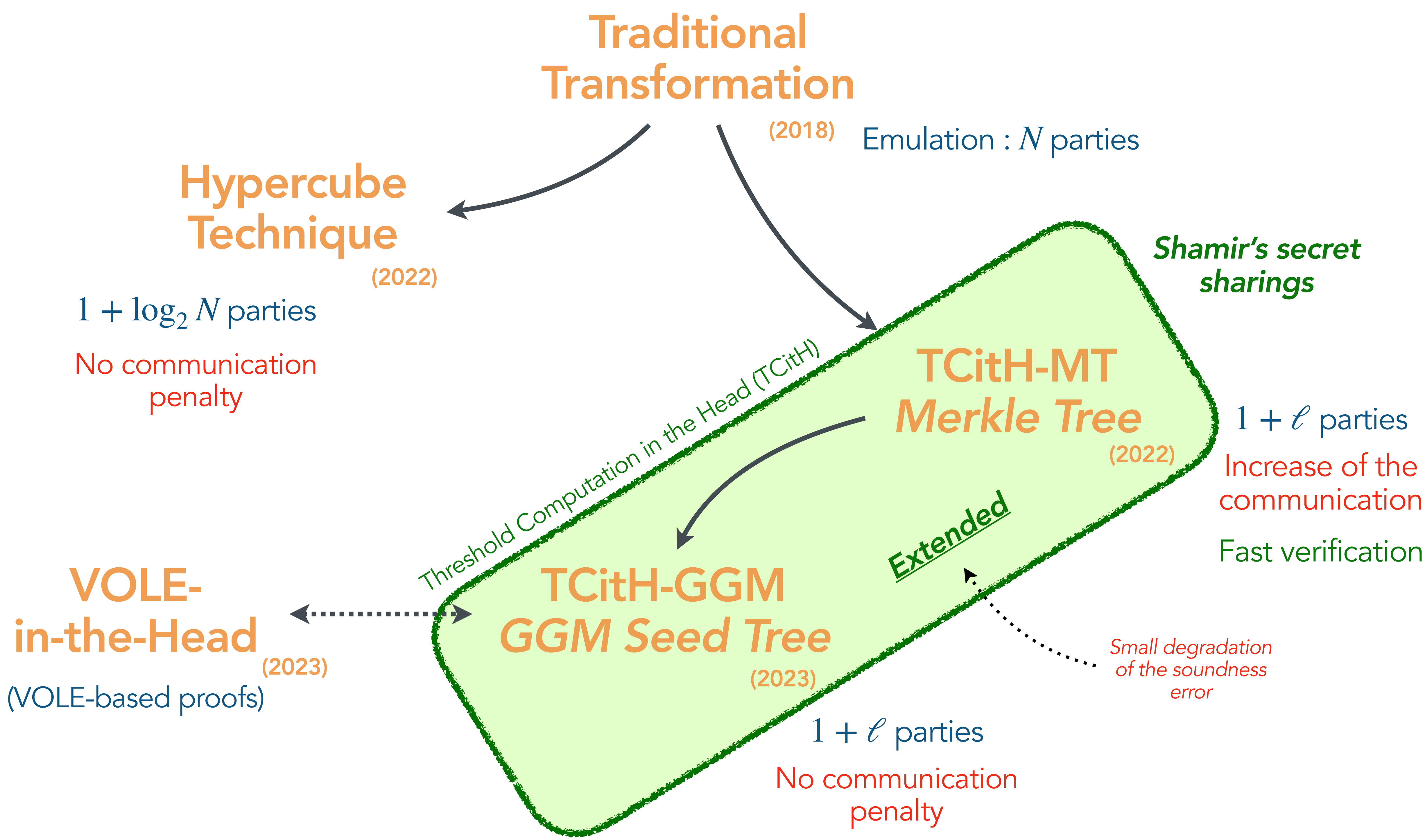

[BBDKORS23] Baum, Braun, Delpech, Klooß, Orsini, Roy, Scholl: "Publicly Verifiable Zero-Knowledge and Post-Quantum Signatures and VOLE-in-the-Head" (Crypto 2023)

Extended TCitH: some applications

[FR23] Feneuil, Rivain: "Threshold Computation in the Head: Improved Framework for Post-Quantum Signatures and Zero-Knowledge Arguments" (ePrint 2023/1573)

Extended TCitH: some applications

- More efficient signature schemes
  - *Unstructured multivariate quadratic (MQ) problem over* $\mathbb{F}_{251}$
    - MQOM: **6.5 KB**
    - Extended TCitH: **4.2 KB**

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Extended TCitH: some applications

- More efficient signature schemes
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    - MQOM: **6.5 KB**
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- Shorter post-quantum ring signature schemes
  - Extended TCitH with MQ: **5.8 KB** in around 8 ms, for 4000 users
  - Extended TCitH with SD: **10.30 KB** in around 10 ms, for 4000 users

[FR23] Feneuil, Rivain: "Threshold Computation in the Head: Improved Framework for Post-Quantum Signatures and Zero-Knowledge Arguments" (ePrint 2023/1573)

Conclusion

- MPC-in-the-Head
  - Very versatile and tunable
  - A practical tool to build *conservative* signature schemes
    - *Between 4-10 KB in few milliseconds (NIST Level I)*

- Perspectives
  - Active research field
  - Signatures with advanced functionalities:

ring signatures, threshold signatures, multi-signatures,

blind signatures, …

PhD Defense

- Title:

  Post-Quantum Signatures
  from Secure Multiparty Computation

- When: Monday 23rd October 2023, at 2 pm
- Where: Sorbonne University (Jussieu)

More information at

https://www.thibauld-feneuil.fr/phd-defense.html