Rank Metric in the Head

RYDE

*N. Aragon, M. Bardet, L. Bidoux,
J.-J. Chi-Domínguez, V. Dyseryn,
T. Feneuil, P. Gaborit, A. Joux,
M. Rivain, J.-P. Tillich, A. Vincotte*

MIRA

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M. Rivain, J.-P. Tillich*

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- MPC-in-the-Head: general principle
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- From MPC-in-the-Head to signatures
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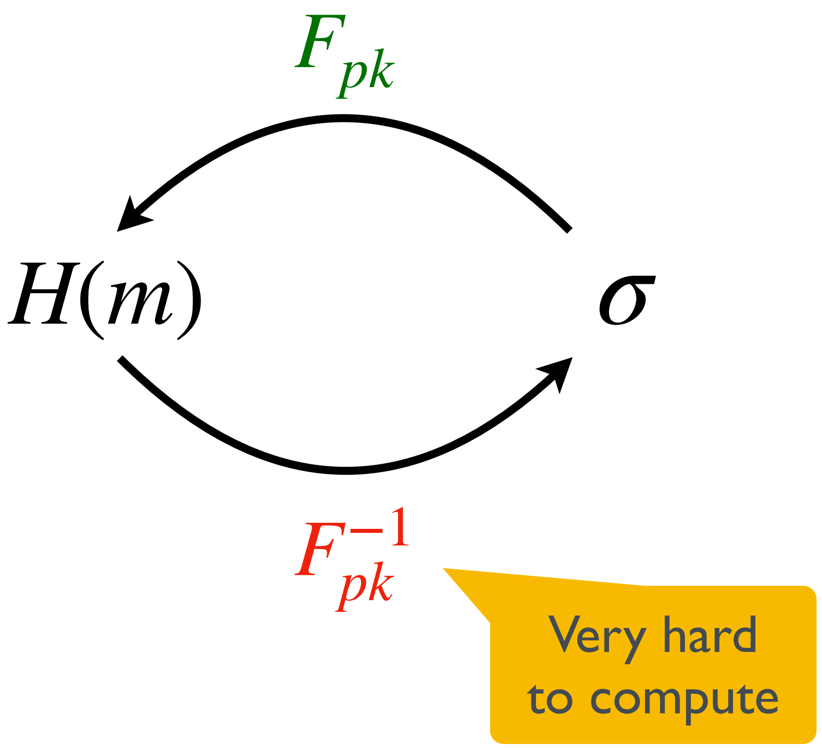
Introduction

How to build signature schemes?

Hash & Sign



- Short signatures
- "Trapdoor" in the public key

How to build signature schemes?

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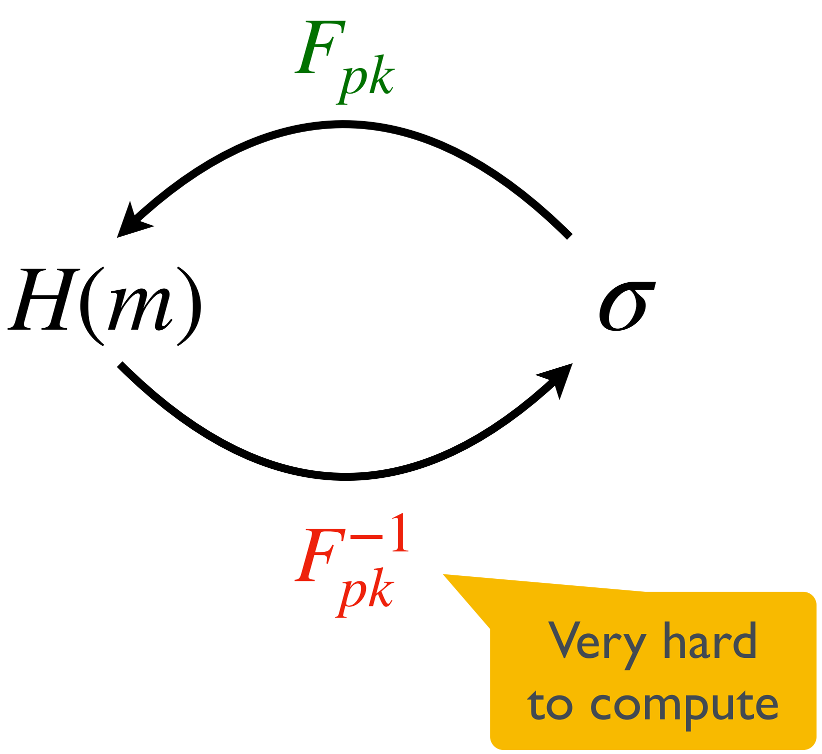
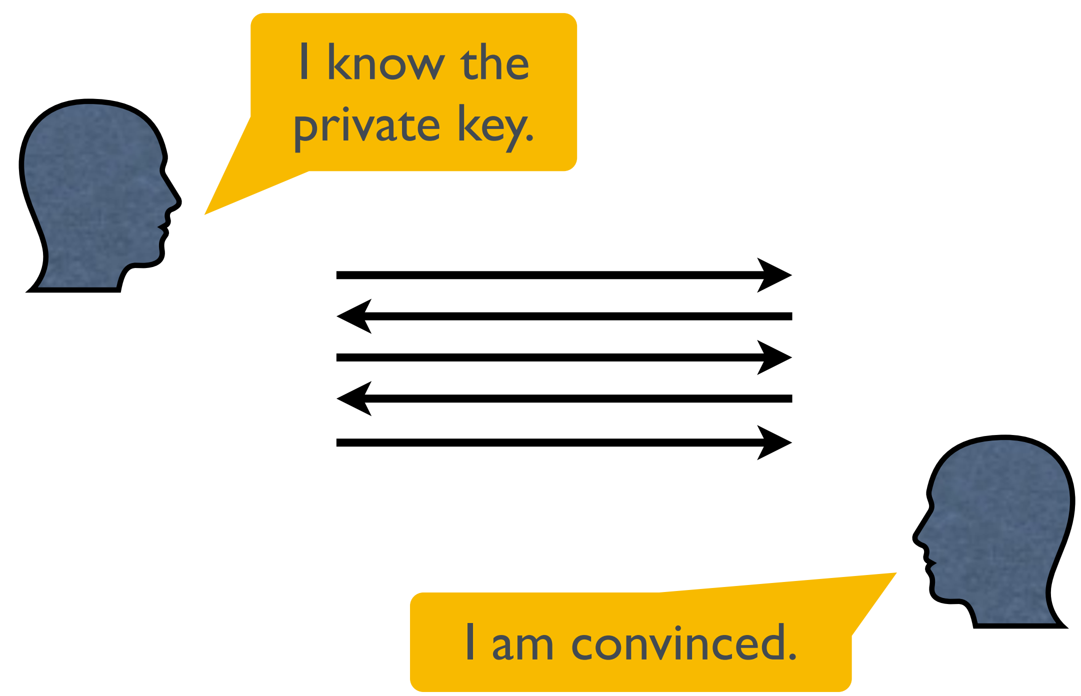


- Short signatures
- “Trapdoor” in the public key

From a zero-knowledge proof



- Large(r) signatures
- Short public key

How to build signature schemes?

Hash & Sign

- Short signatures
- "Trapdoor" in the public key

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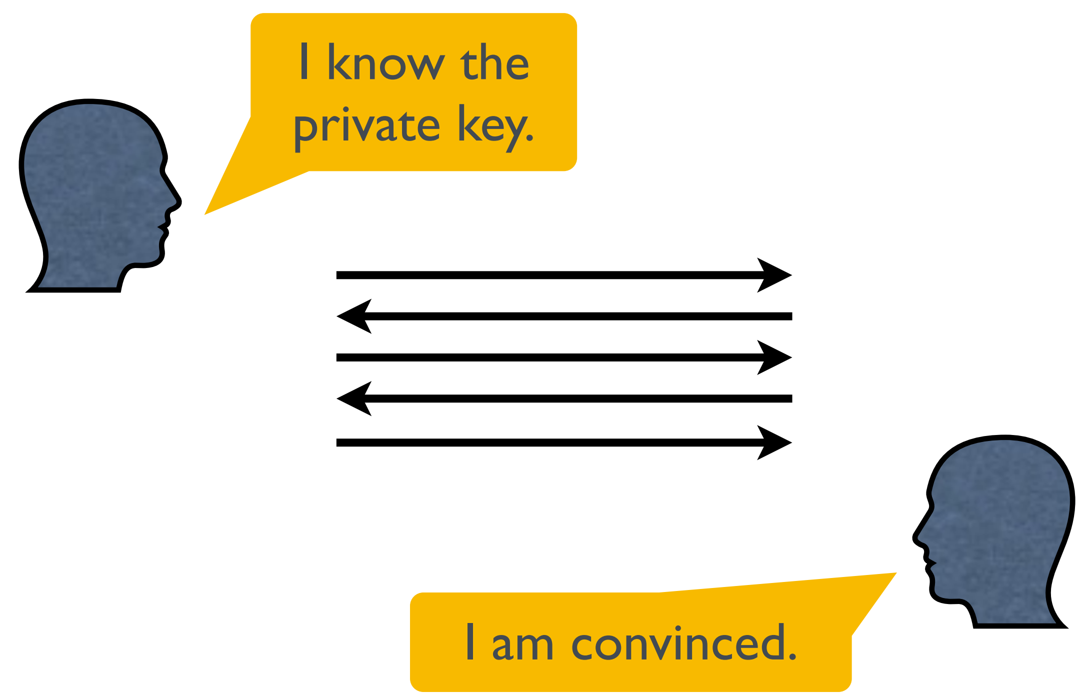

- Large(r) signatures
- Short public key

Proof of knowledge

- **Completeness:** Pr[verif ✓ | honest prover] = 1
- **Soundness:** Pr[verif ✓ | malicious prover] $\leq \varepsilon$ (*e.g.* 2^{-128})
- **Zero-knowledge:** verifier learns nothing on x

MPC in the Head

- **[IKOS07]** Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, Amit Sahai: "Zero-knowledge from secure multiparty computation" (STOC 2007)
- Turn an MPC protocol into a zero knowledge proof of knowledge
- **Generic**: can be applied to any cryptographic problem

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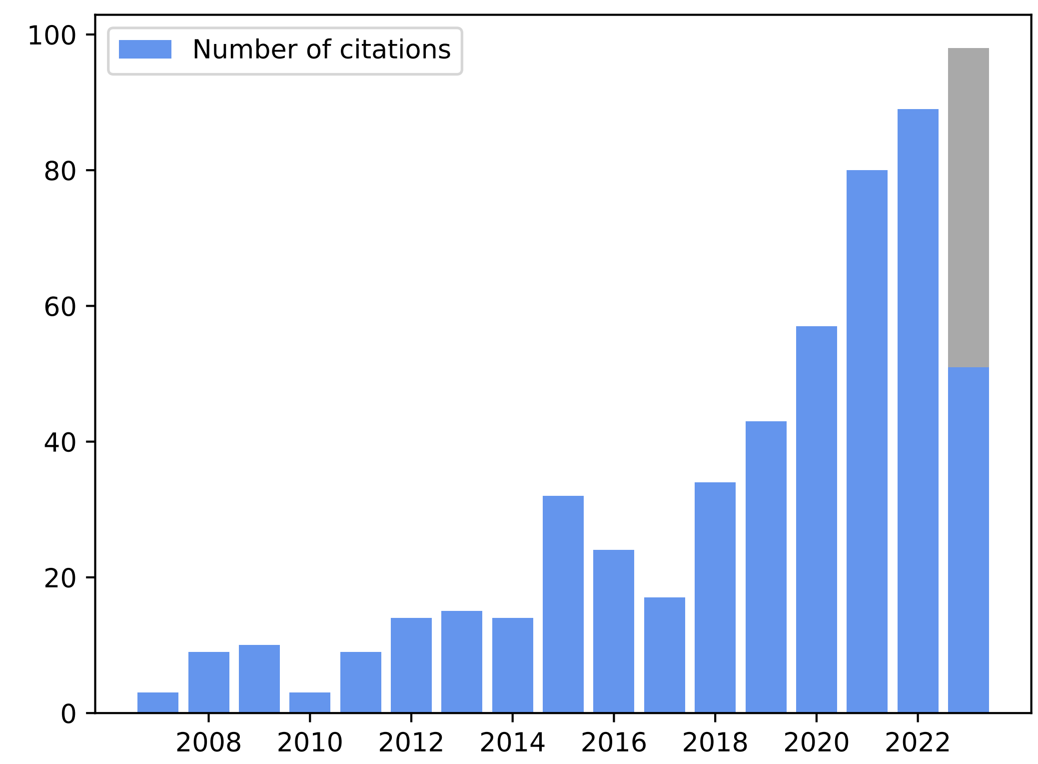

Figure: Number of citations to [IKOS07] by year

Source: Google Scholar

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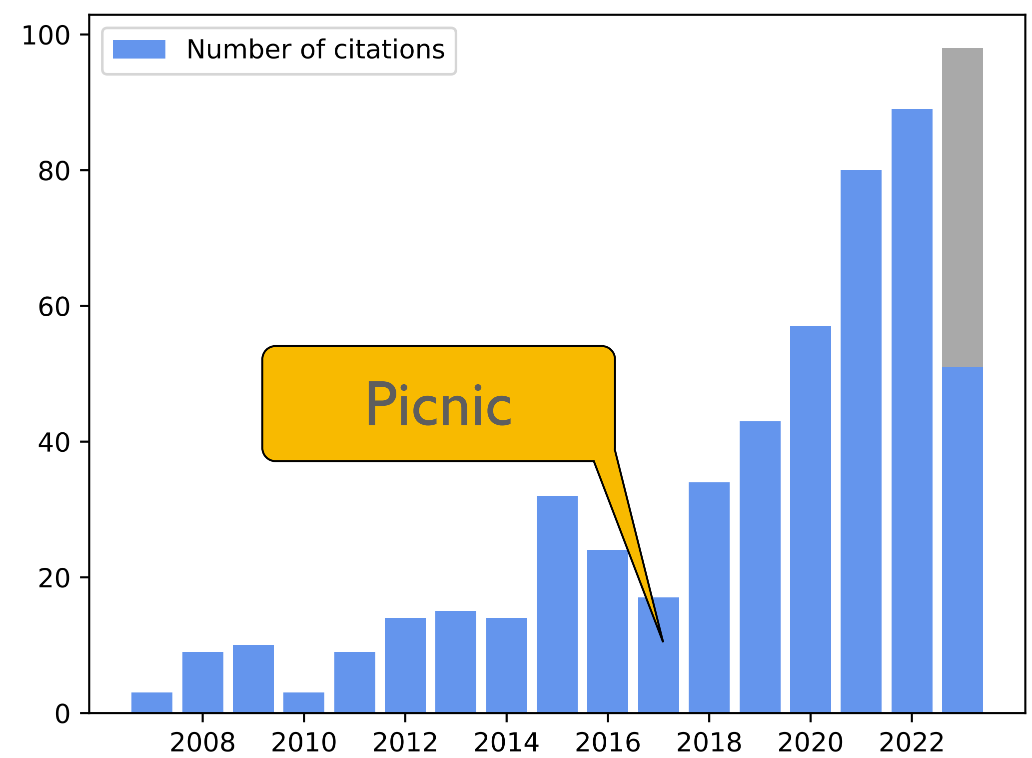

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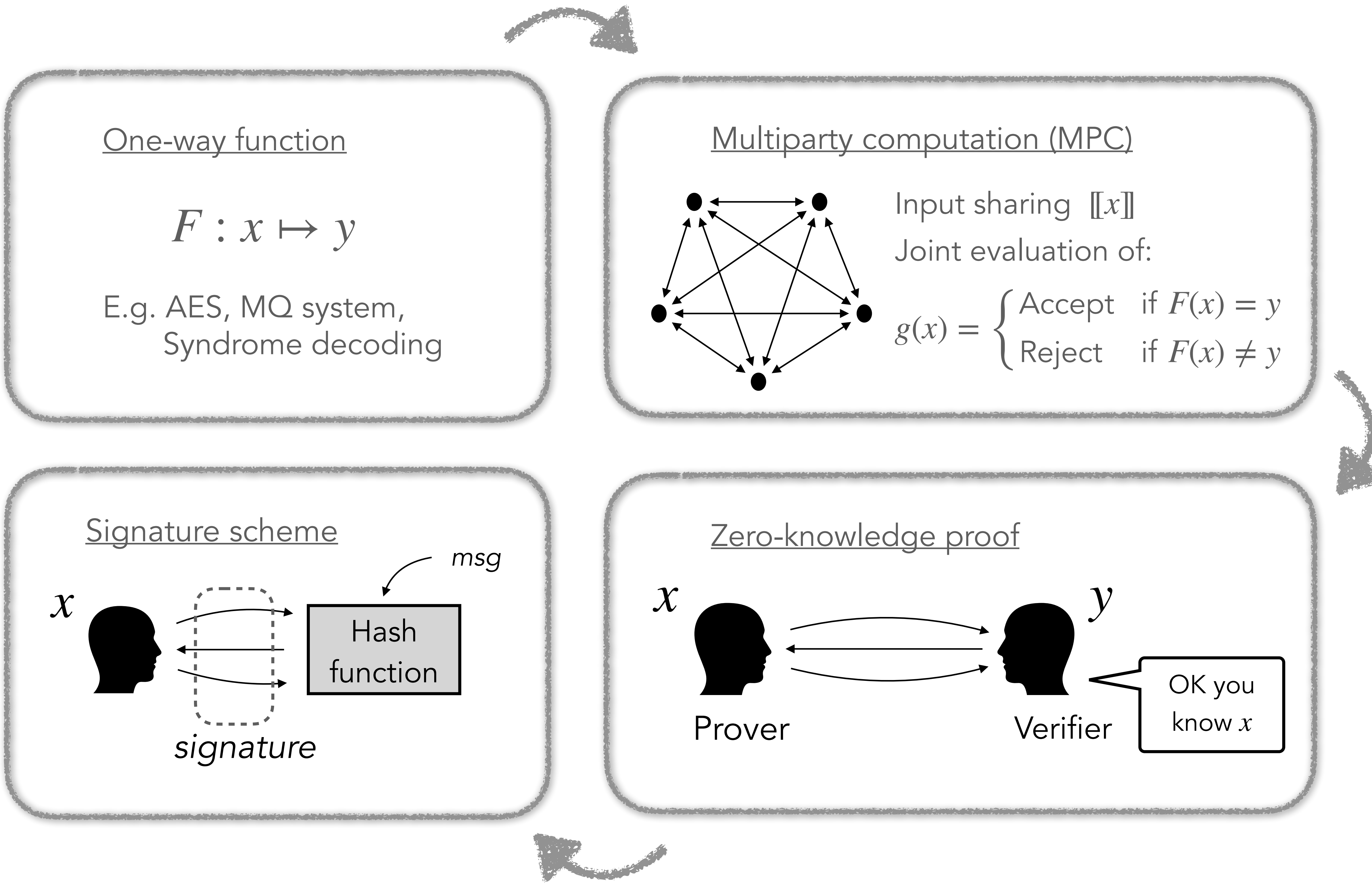
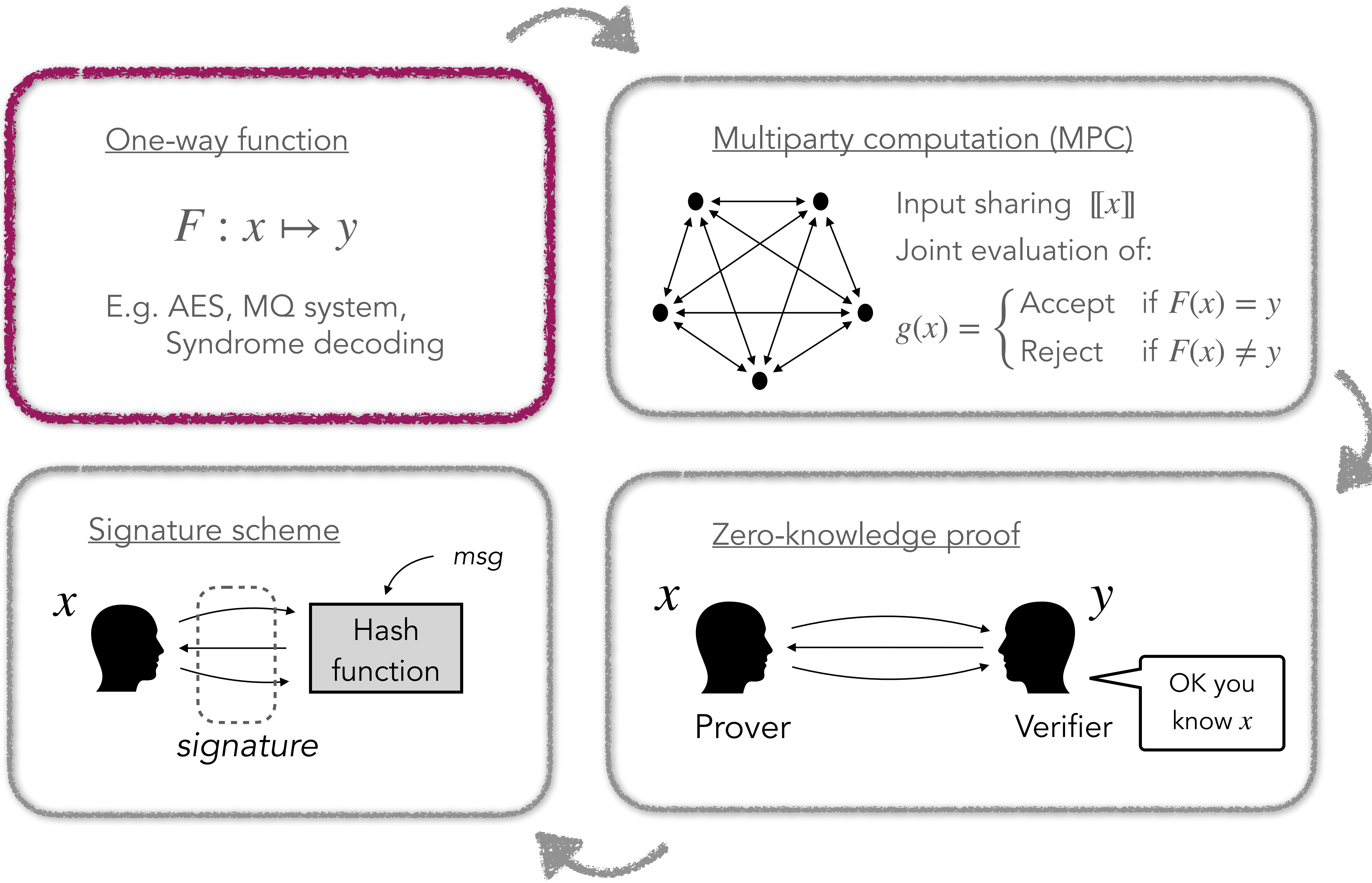
- **[IKOS07]** Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, Amit Sahai: "Zero-knowledge from secure multiparty computation" (STOC 2007)
- Turn an MPC protocol into a zero knowledge proof of knowledge
- **Generic**: can be applied to any cryptographic problem
- Convenient to build (candidate) **post-quantum signature** schemes
- **Picnic**: submission to NIST (2017)
- First round of recent NIST call: 8 MPCitH schemes / 40 submissions

AIMer	*MQOM*
Biscuit	*PERK*
MIRA	*RYDE*
MiRitH	*SDitH*



One-way function
$F : x \mapsto y$
E.g. AES, MQ system,
Syndrome decoding
Multiparty computation (MPC)
Input sharing $[\![x]\!]$
Joint evaluation of:
$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$
Signature scheme
msg
x
Hash function
signature
Zero-knowledge proof
x
y
Prover
Verifier
OK you know x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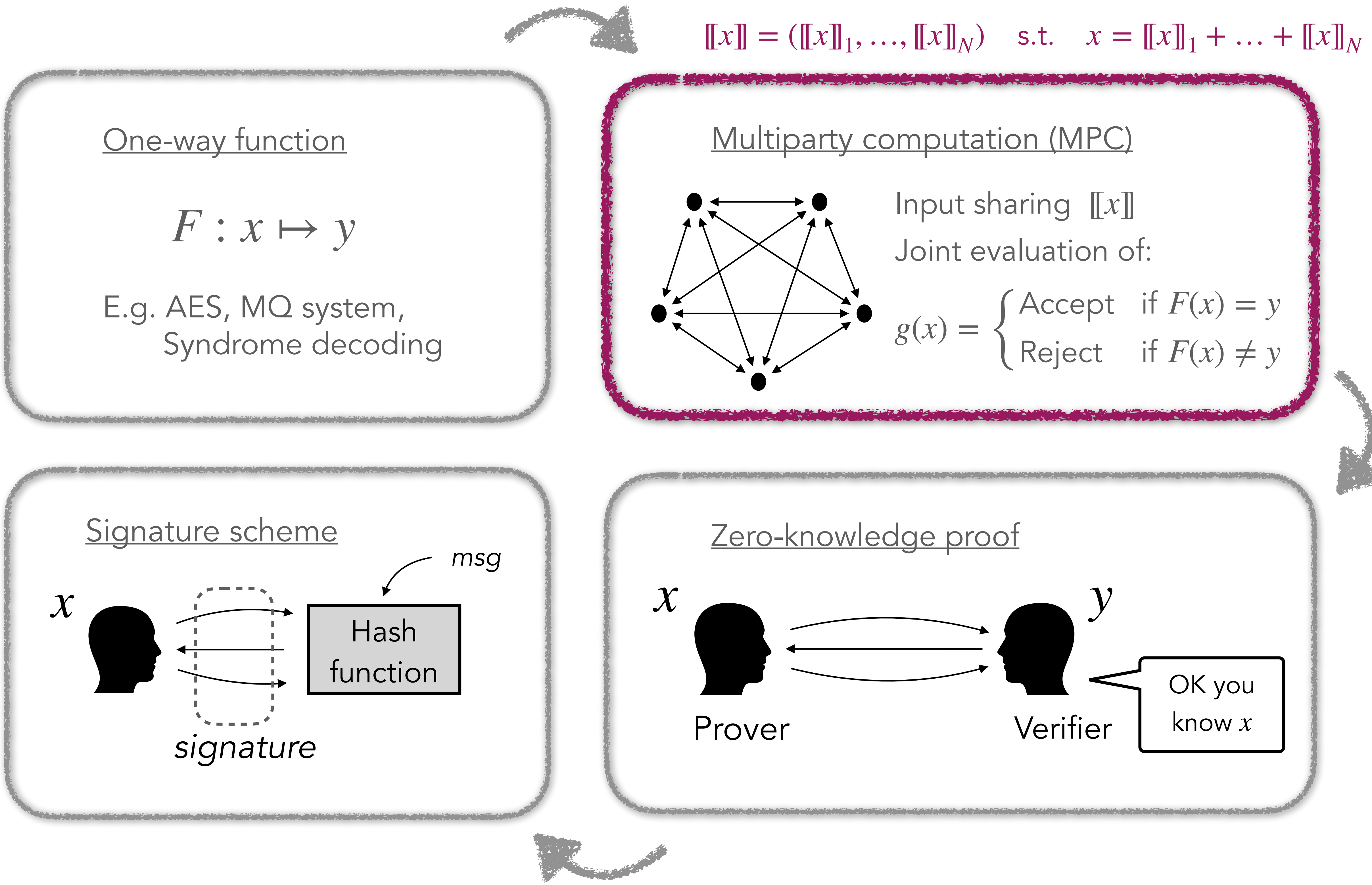

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$ s.t. $x = [\![x]\!]_1 + \ldots + [\![x]\!]_N$
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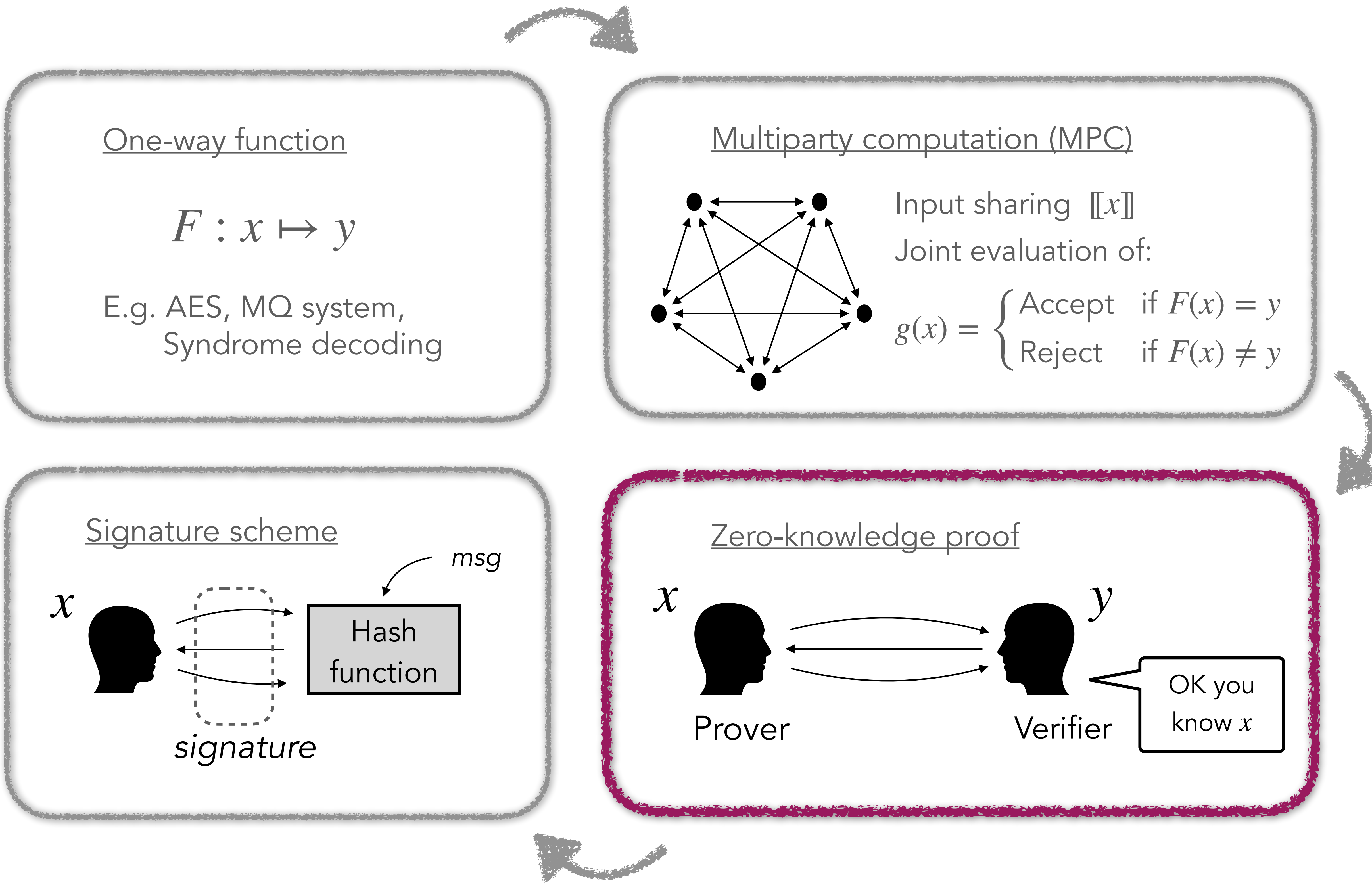

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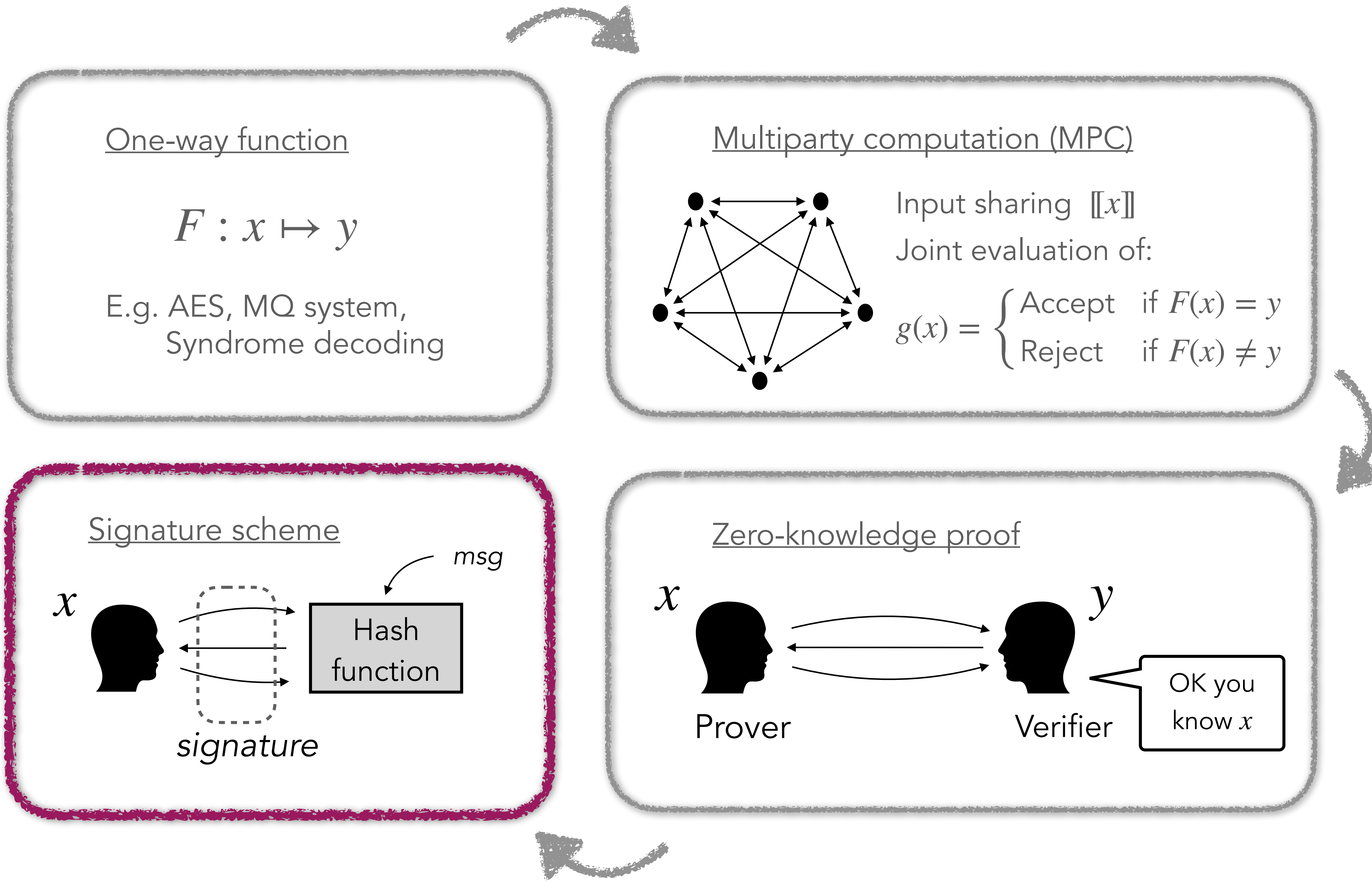

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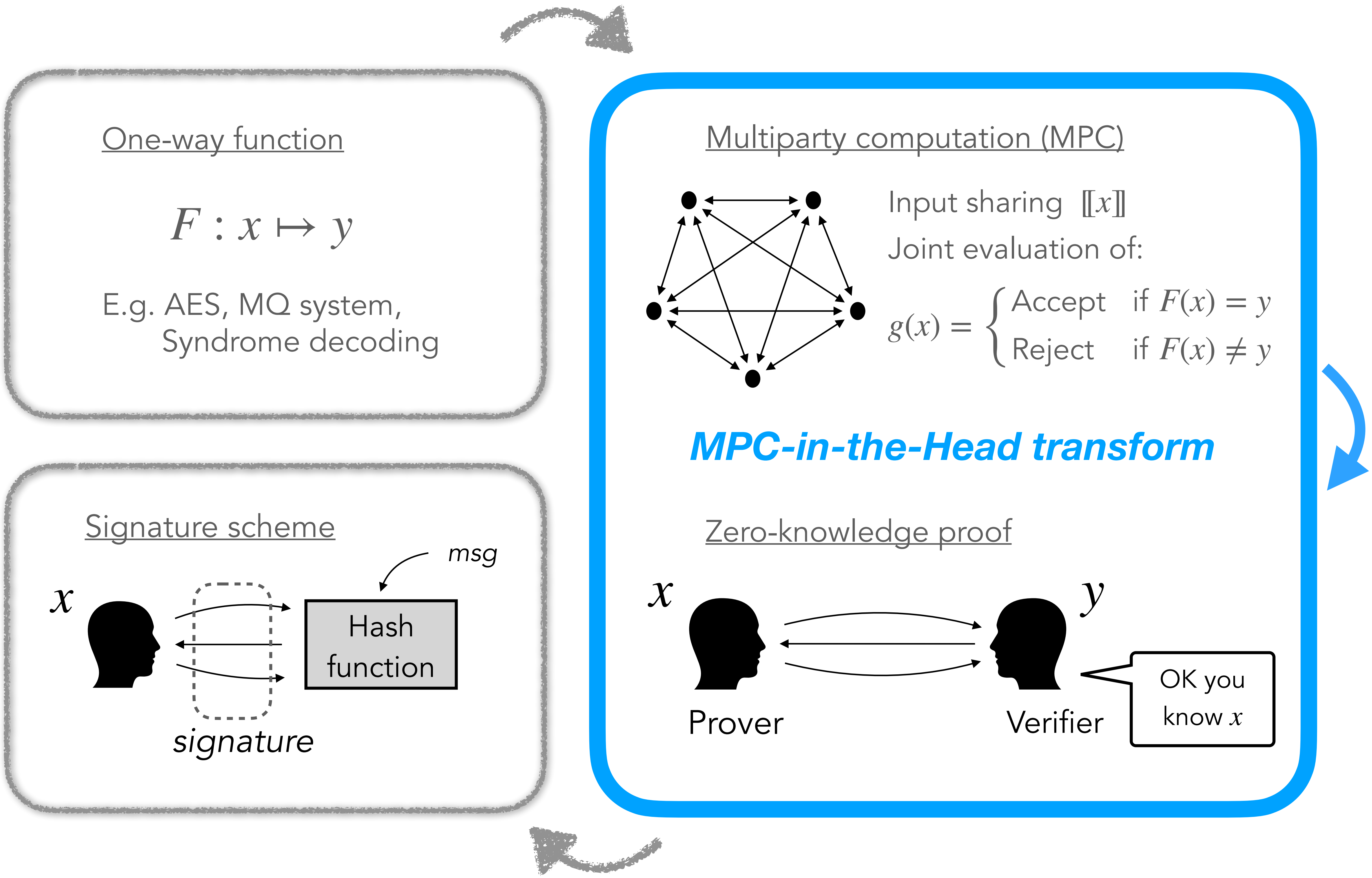
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MPC-in-the-Head transform
Zero-knowledge proof
x
y
Prover
Verifier
OK you
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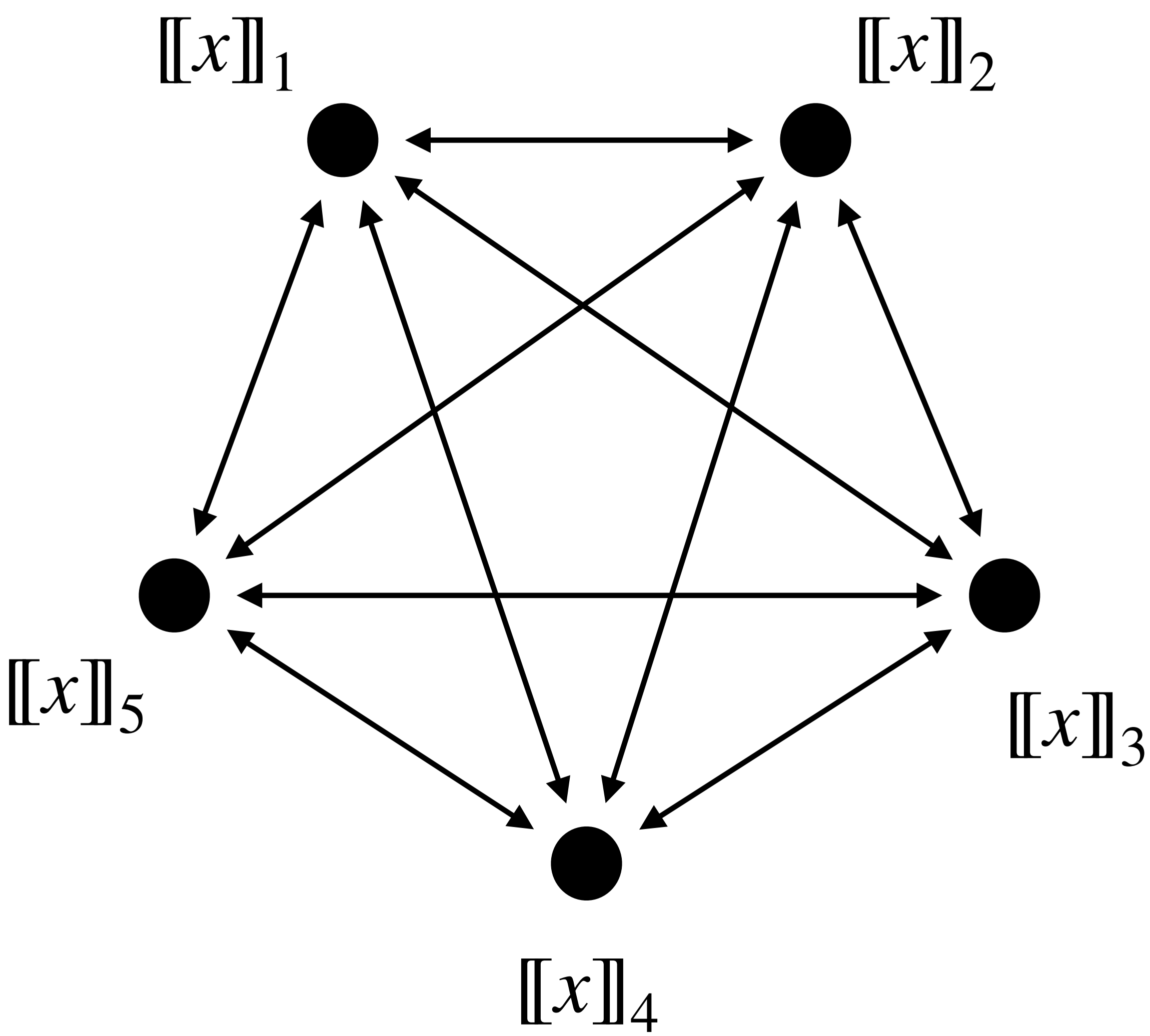
MPCitH: general principle

MPC model



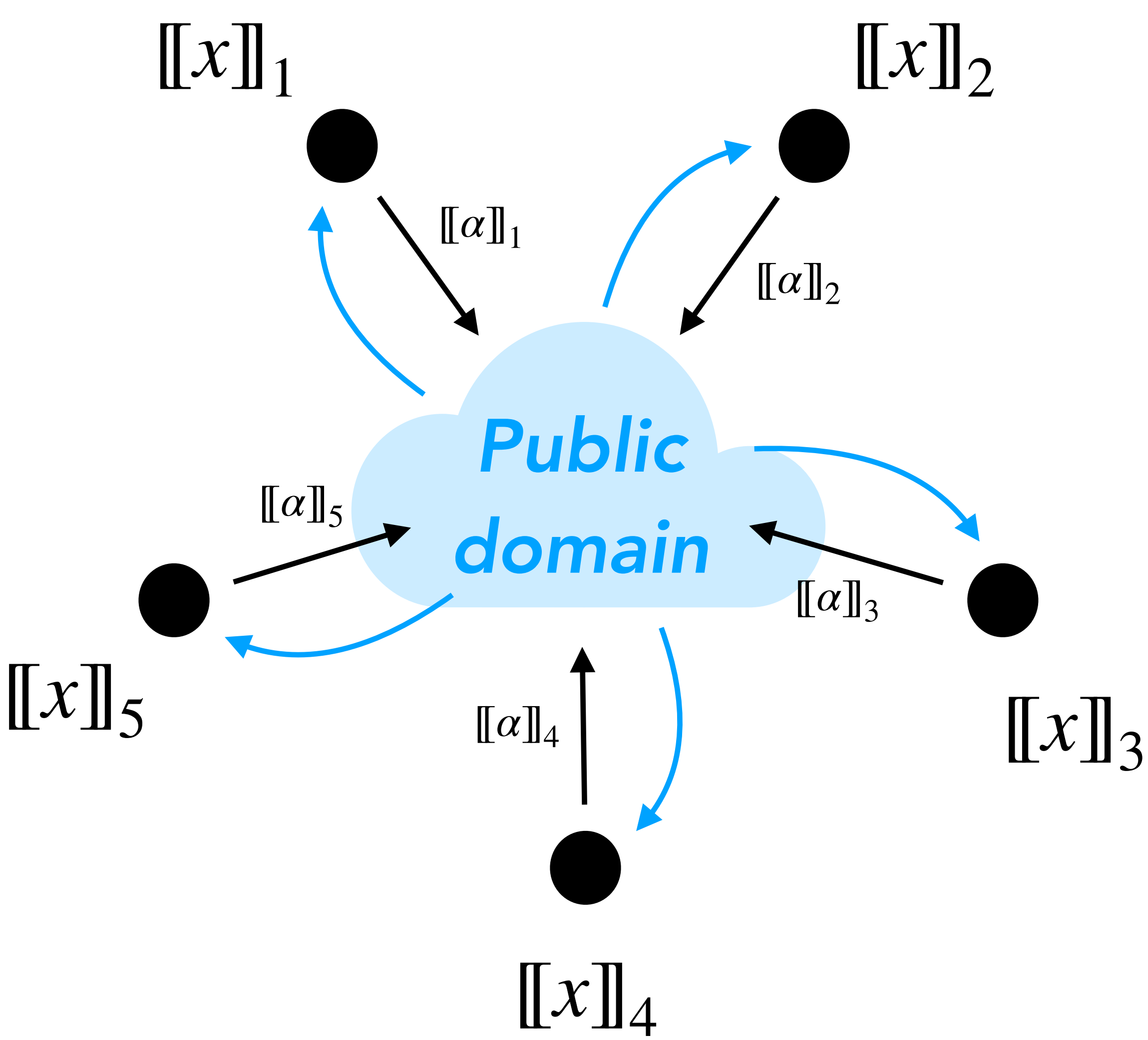
$$x = [\![x]\!]_1 + [\![x]\!]_2 + \ldots + [\![x]\!]_N$$

- **Jointly compute**

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

- $(N-1)$ **private:** the views of any $N-1$ parties provide no information on x
- **Semi-honest model:** assuming that the parties follow the steps of the protocol

MPC model



$$x = [\![x]\!]_1 + [\![x]\!]_2 + \ldots + [\![x]\!]_N$$

- **Jointly compute**

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

- $(N-1)$ **private:** the views of any $N-1$ parties provide no information on x
- **Semi-honest model:** assuming that the parties follow the steps of the protocol
- ***Broadcast model***
  - *Parties locally compute on their shares* $[\![x]\!] \mapsto [\![\alpha]\!]$
  - *Parties broadcast* $[\![\alpha]\!]$ *and recompute* α
  - *Parties start again (now knowing* α*)*

MPCitH transform

Prover

Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \dots, [\![x]\!]_N)$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\cdots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

Prover

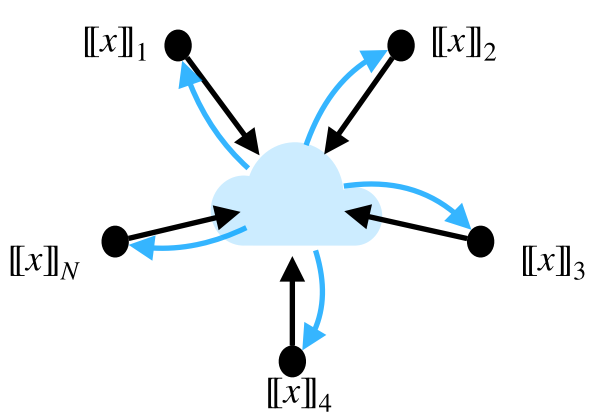
Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

② Run MPC in their head



$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

...

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

Prover

Verifier

MPCitH transform

① Generate and commit shares

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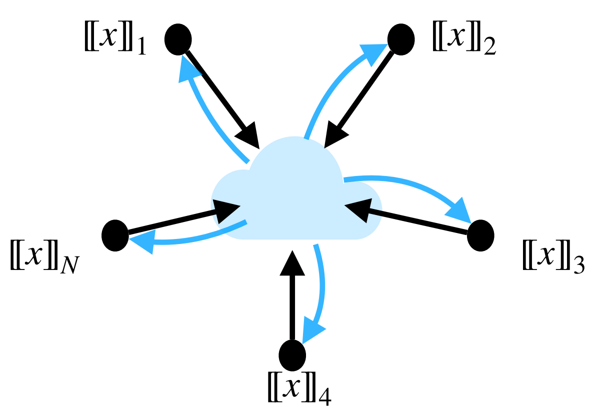


$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

...

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

i^*

③ Choose a random party

$i^* \xleftarrow{\$} \{1, \ldots, N\}$

Prover

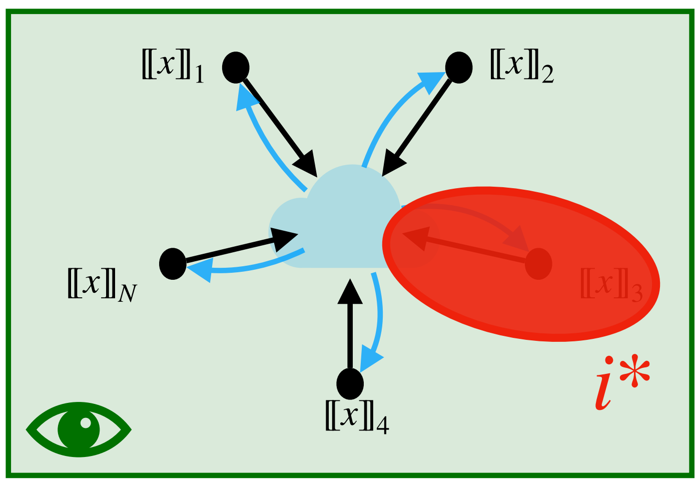
Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

② Run MPC in their head



④ Open parties $\{1,\ldots,N\}\backslash\{i^*\}$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\cdots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$ →

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$ →

← i^*

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$ →

③ Choose a random party

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Verifier

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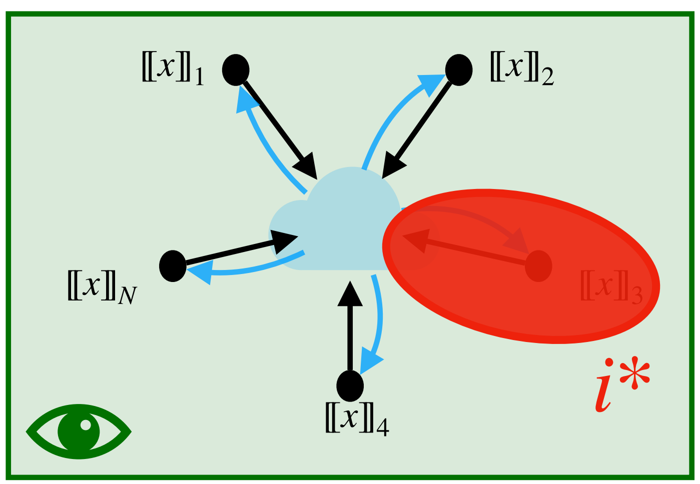

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$i^* \xleftarrow{\$} \{1, \ldots, N\}$

⑤ Check $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $\tilde{g}(y, \alpha) = \mathrm{Accept}$

Prover

Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

We have $F(x) \neq y$ *where*

$x := [\![x]\!]_1 + \ldots + [\![x]\!]_N$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\ldots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

→

Malicious Prover

Verifier

MPCitH transform

① Generate and commit shares

$$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$$

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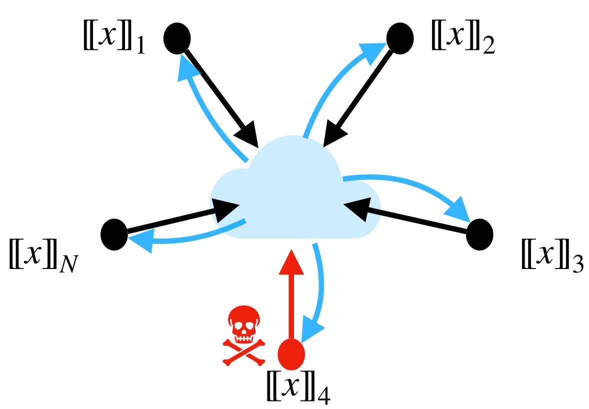
② Run MPC in their head



$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\cdots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

→

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

→

Malicious Prover

Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

We have $F(x) \neq y$ where

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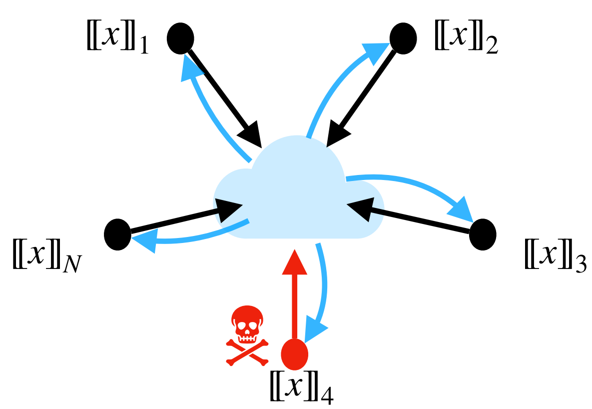
② Run MPC in their head



$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\ldots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

③ Choose a random party

$i^* \xleftarrow{\$} \{1, \ldots, N\}$

i^*

Malicious Prover

Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

We have $F(x) \neq y$ where

$x := [\![x]\!]_1 + \ldots + [\![x]\!]_N$

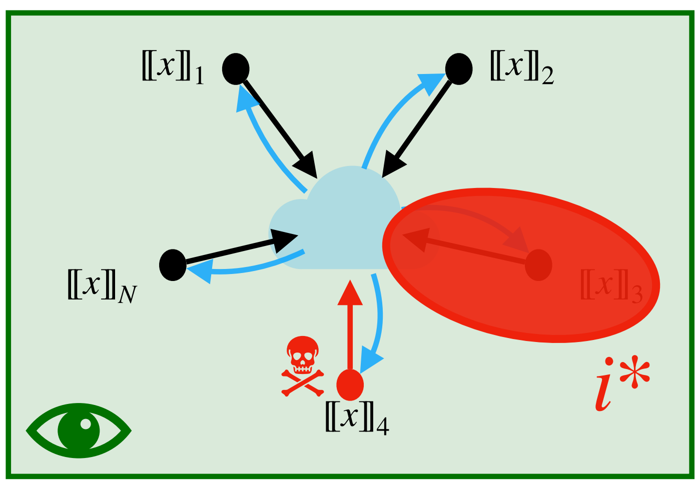
$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

...

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

→

② Run MPC in their head



send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

→

③ Choose a random party

$i^* \xleftarrow{\$} \{1, \ldots, N\}$

i^*

←

④ Open parties $\{1, \ldots, N\} \backslash \{i^*\}$

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$

→

Malicious Prover

Verifier

MPCitH transform

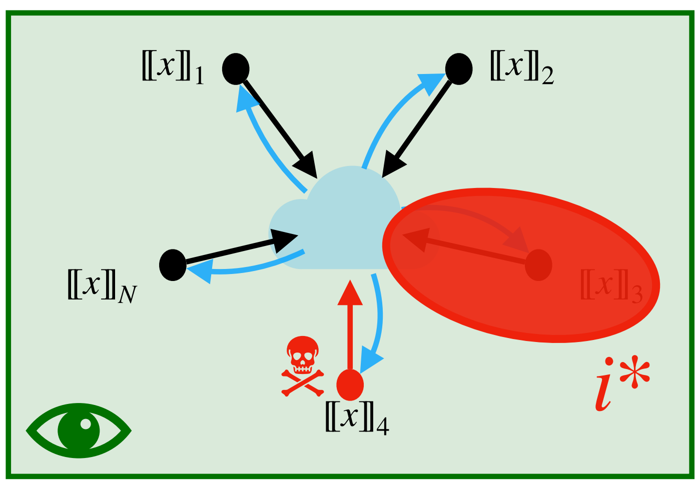
① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

We have $F(x) \neq y$ where

$x := [\![x]\!]_1 + \ldots + [\![x]\!]_N$

② Run MPC in their head



④ Open parties $\{1, \ldots, N\} \backslash \{i^*\}$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\ldots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$ →

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$ →

← i^*

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$ →

③ Choose a random party

$i^* \xleftarrow{\$} \{1, \ldots, N\}$

⑤ Check $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $\tilde{g}(y, \alpha) =$ Accept

Malicious Prover

Verifier

Cheating detected!

MPCitH transform

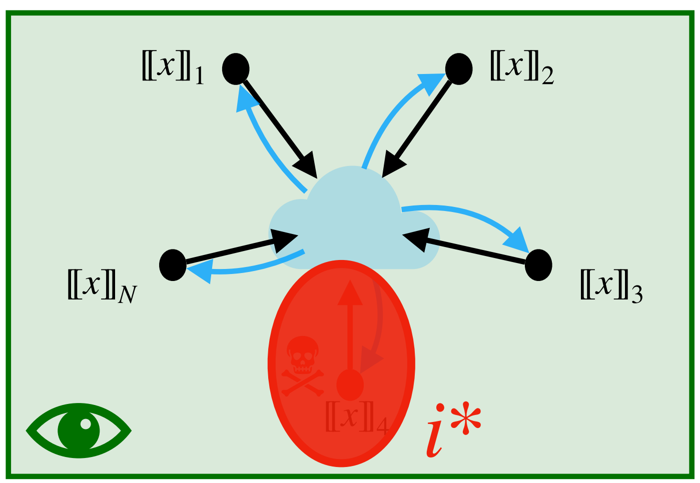
① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

We have $F(x) \neq y$ where

$x := [\![x]\!]_1 + \ldots + [\![x]\!]_N$

② Run MPC in their head



④ Open parties $\{1,\ldots,N\} \backslash \{i^*\}$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\cdots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

→

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

→

i^*

←

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$

→

③ Choose a random party

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⑤ Check $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $\tilde{g}(y, \alpha) =$ Accept

Malicious Prover

Verifier

✓ **Seems OK.**

MPCitH transform

- **Zero-knowledge** $\iff$ MPC protocol is $(N-1)$-private

MPCitH transform

- **Zero-knowledge** $\iff$ MPC protocol is $(N-1)$-private

- **Soundness:**

$$
\begin{aligned}
&\mathbb{P}(\text{malicious prover convinces the verifier}) \\
&\qquad = \mathbb{P}(\text{corrupted party remains hidden}) \\
&\qquad = \frac{1}{N}
\end{aligned}
$$

MPCitH transform

- **Zero-knowledge** $\iff$ MPC protocol is $(N-1)$-private
- **Soundness:**

$$\mathbb{P}(\text{malicious prover convinces the verifier}) = \mathbb{P}(\text{corrupted party remains hidden}) = \frac{1}{N}$$

- **Parallel repetition**

Protocol repeated τ times in parallel → soundness error $\left(\frac{1}{N}\right)^{\tau}$

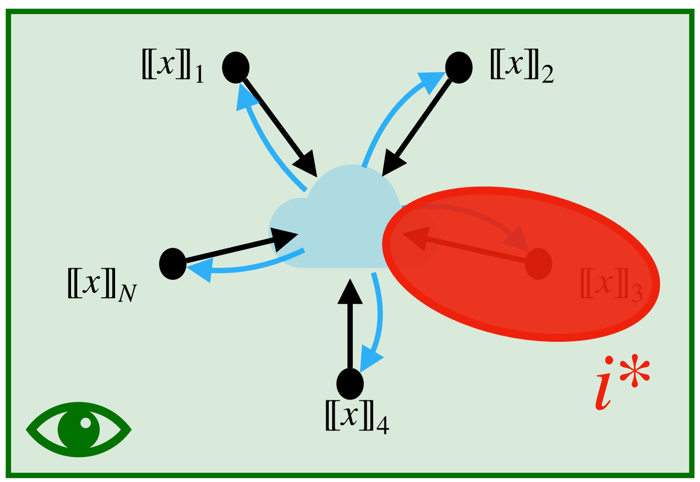
Optimisations

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

② Run MPC in their head



④ Open parties $\{1,\ldots,N\}\backslash\{i^*\}$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

...

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

→

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

→

③ Choose a random party

$i^* \xleftarrow{\$} \{1,\ldots,N\}$

i^*

←

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$

→

⑤ Check $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $\tilde{g}(y, \alpha) = \text{Accept}$

<u>Prover</u> <u>Verifier</u>

Naive MPCitH transformation

Size of the broadcast (per party)

Size of a commitment digest

Size of the MPC input (per party)

$$\text{Size} \approx \tau \cdot \left(N \cdot 2\lambda + N \cdot |\alpha| + (N-1) \cdot |x| \right)$$

Number of repetitions to achieve the desired security level

$$\tau \approx \frac{\lambda}{\log_2 N}$$

Naive MPCitH transformation

Size of the broadcast (per party)

Size of a commitment digest

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Number of repetitions to achieve the desired security level

$$\tau \approx \frac{\lambda}{\log_2 N}$$

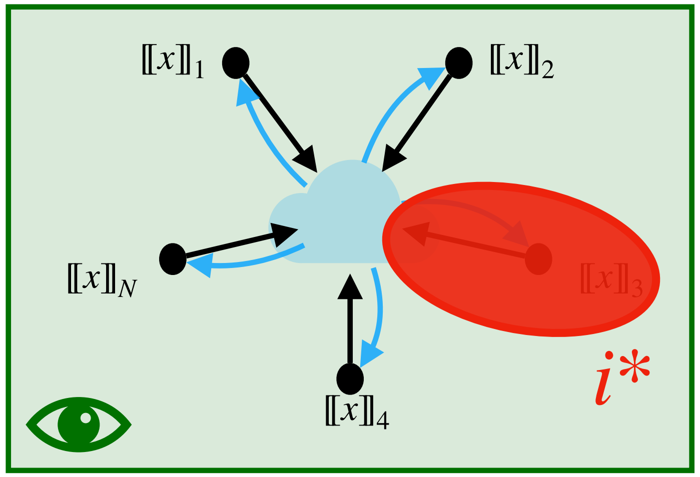
The signature sizes would be of at least 30 KB.

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

② Run MPC in their head



④ Open parties $\{1, \ldots, N\} \backslash \{i^*\}$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

...

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

i^*

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$

③ Choose a random party

$i^* \leftarrow^{\$} \{1, \ldots, N\}$

⑤ Check $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $\tilde{g}(y, \alpha) = \text{Accept}$

Prover

Verifier

MPCitH transform

① Generate and commit shares

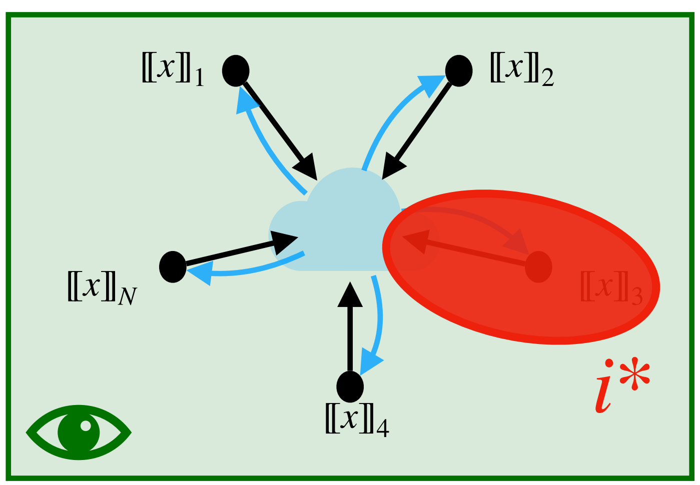
$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

Compute

$\forall i, \mathsf{com}_i = \mathrm{Com}^{\rho_i}([\![x]\!]_i)$

$h_1 = \mathrm{Hash}(\mathsf{com}_1, \ldots, \mathsf{com}_N)$ →

② Run MPC in their head



$h_2 = \mathrm{Hash}([\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N)$ →

③ Choose a random party

$i^* \xleftarrow{\$} \{1, \ldots, N\}$

← i^*

④ Open parties $\{1, \ldots, N\} \backslash \{i^*\}$

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$ $(\mathsf{com}_{i^*}, \alpha)$ →

⑤ Compute $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $\tilde{g}(y, \alpha) = \mathsf{Accept}$

Check $h_1 = \mathrm{Hash}(\mathsf{com}_1, \ldots, \mathsf{com}_N)$

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Verifier

MPCitH transform

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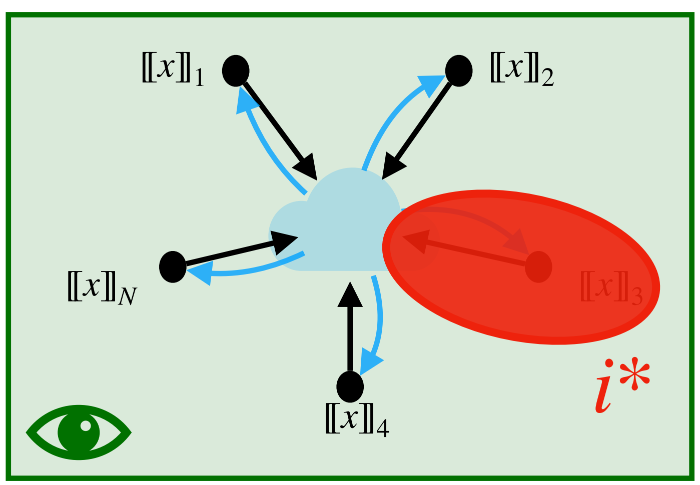

④ Open parties $\{1,\ldots,N\} \backslash \{i^*\}$

$h_1 = \text{Hash}(\text{com}_1, \ldots, \text{com}_N)$ →

$h_2 = \text{Hash}([\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N)$ →

← i^*

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$ $(\text{com}_{i^*}, \alpha)$ →

③ Choose a random party

$i^* \xleftarrow{\$} \{1,\ldots,N\}$

⑤ Compute $\forall i \neq i^*$

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Check $\tilde{g}(y, \alpha) = \text{Accept}$

Check $h_1 = \text{Hash}(\text{com}_1, \ldots, \text{com}_N)$

Check $h_2 = \text{Hash}([\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N)$

<u>Prover</u>

<u>Verifier</u>

Using a Seed Tree

[KKW18] Katz, Kolesnikov, Wang: "Improved Non-Interactive Zero Knowledge with Applications to Post-Quantum Signatures" (CCS 2018)

$$x = [\![x]\!]_1 + [\![x]\!]_2 + [\![x]\!]_3 + \ldots + [\![x]\!]_{N-1} + [\![x]\!]_N$$

Using a Seed Tree

[KKW18] Katz, Kolesnikov, Wang: "Improved Non-Interactive Zero Knowledge with Applications to Post-Quantum Signatures" (CCS 2018)

$$\begin{array}{ccccccccccccc} & \text{seed}_1 & & \text{seed}_2 & & \text{seed}_3 & & & & \text{seed}_{N-1} & & \text{seed}_N \\ & \big\downarrow{\scriptstyle \text{PRG}} & & \big\downarrow{\scriptstyle \text{PRG}} & & \big\downarrow{\scriptstyle \text{PRG}} & & & & \big\downarrow{\scriptstyle \text{PRG}} & & \big\downarrow{\scriptstyle \text{PRG}} \; {\color{red}+\Delta x} \\ x = & [\![x]\!]_1 & + & [\![x]\!]_2 & + & [\![x]\!]_3 & + & \ldots & + & [\![x]\!]_{N-1} & + & [\![x]\!]_N \end{array}$$

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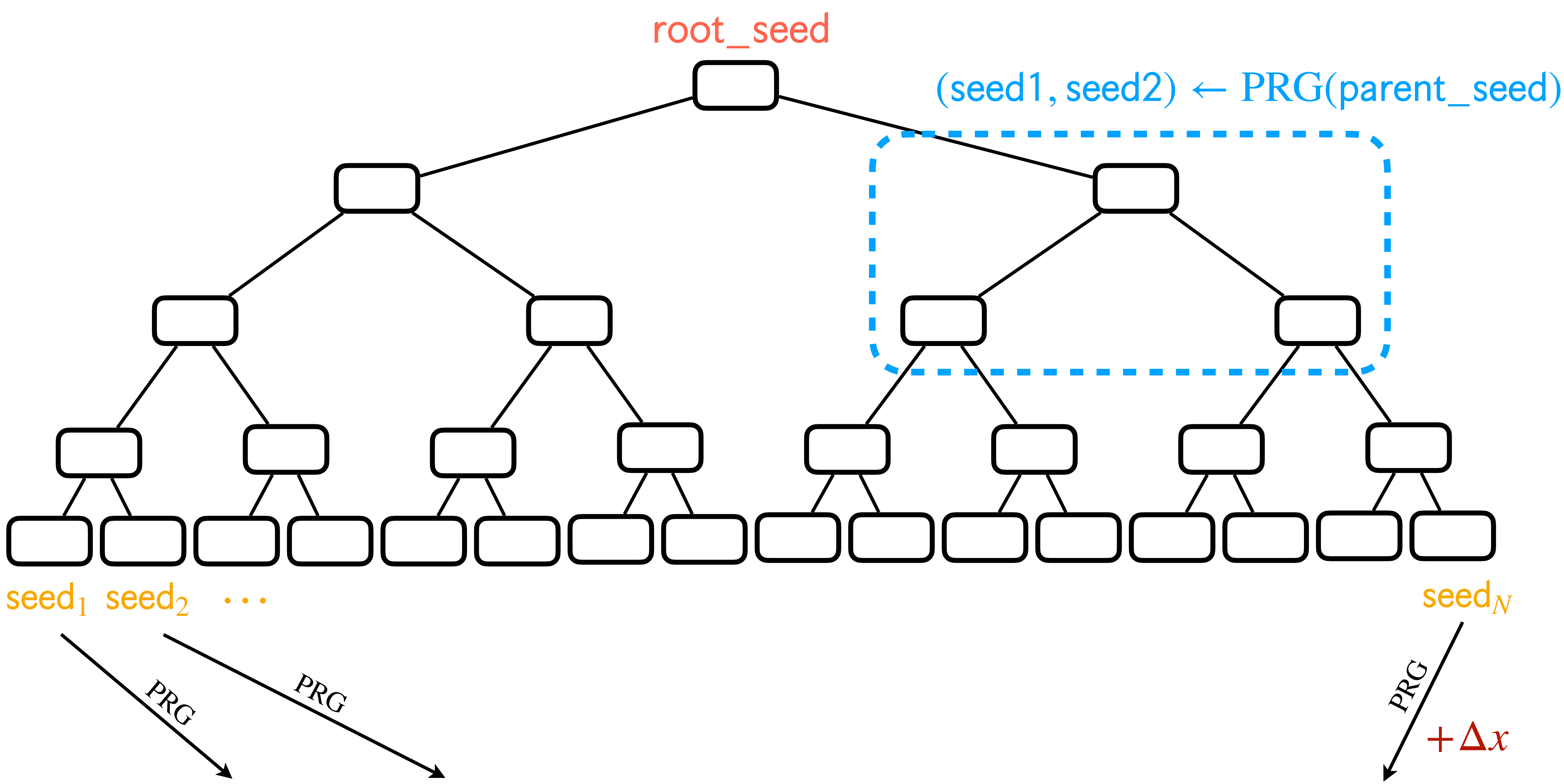

$$x = [\![x]\!]_1 + [\![x]\!]_2 + [\![x]\!]_3 + \ldots + [\![x]\!]_{N-1} + [\![x]\!]_N$$

Using a Seed Tree

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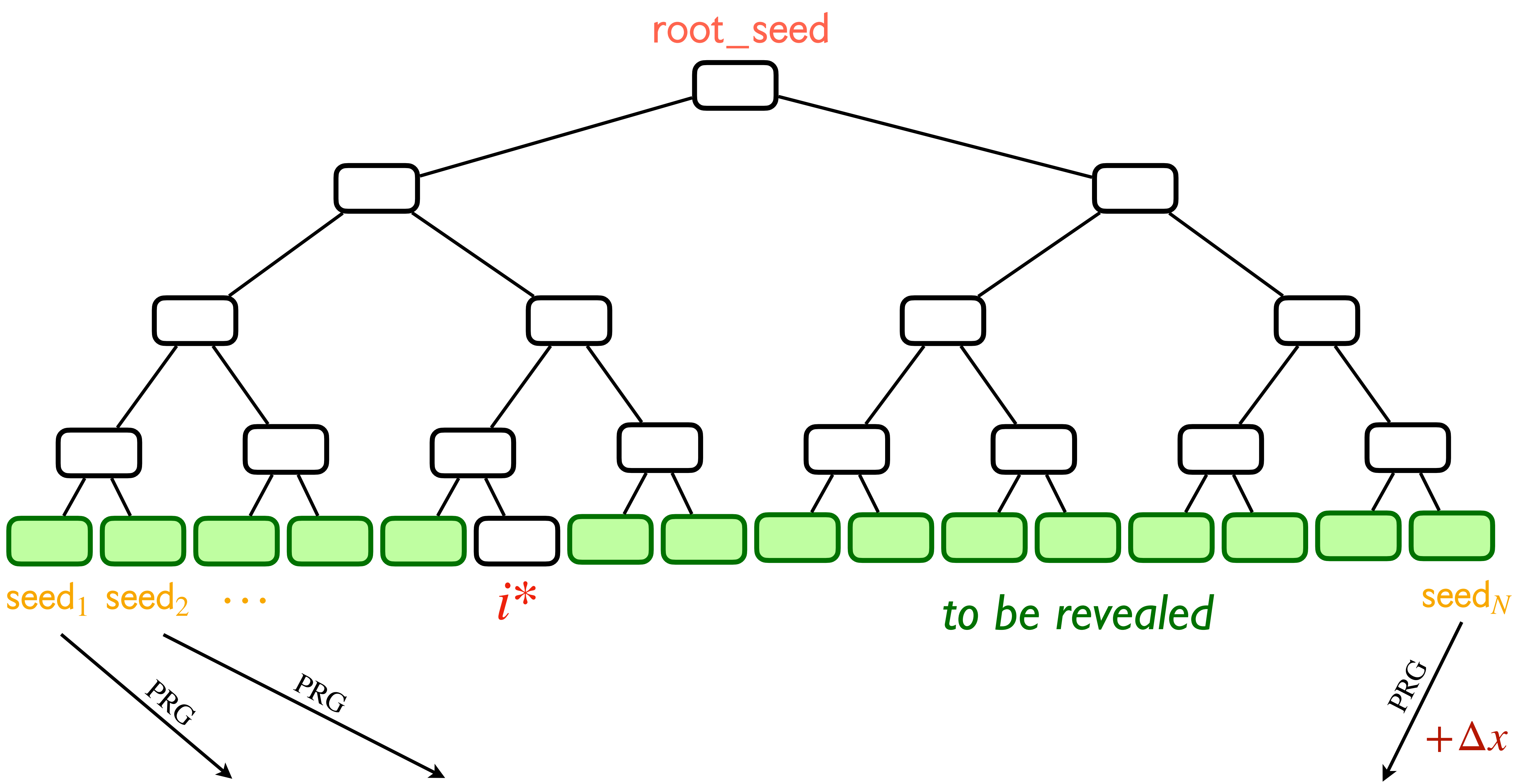

$$x = [\![x]\!]_1 + [\![x]\!]_2 + [\![x]\!]_3 + \ldots + [\![x]\!]_{N-1} + [\![x]\!]_N$$

Using a Seed Tree

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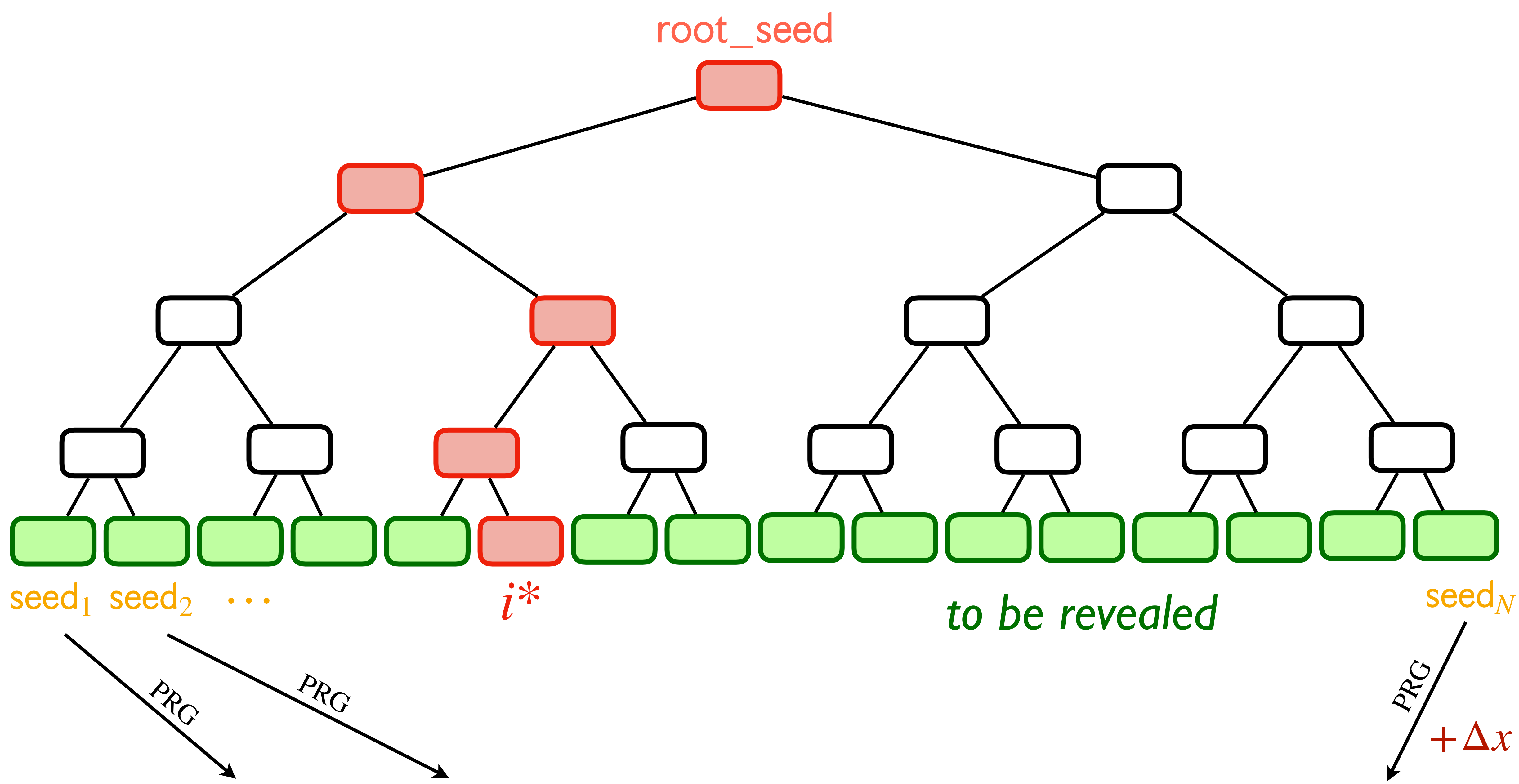

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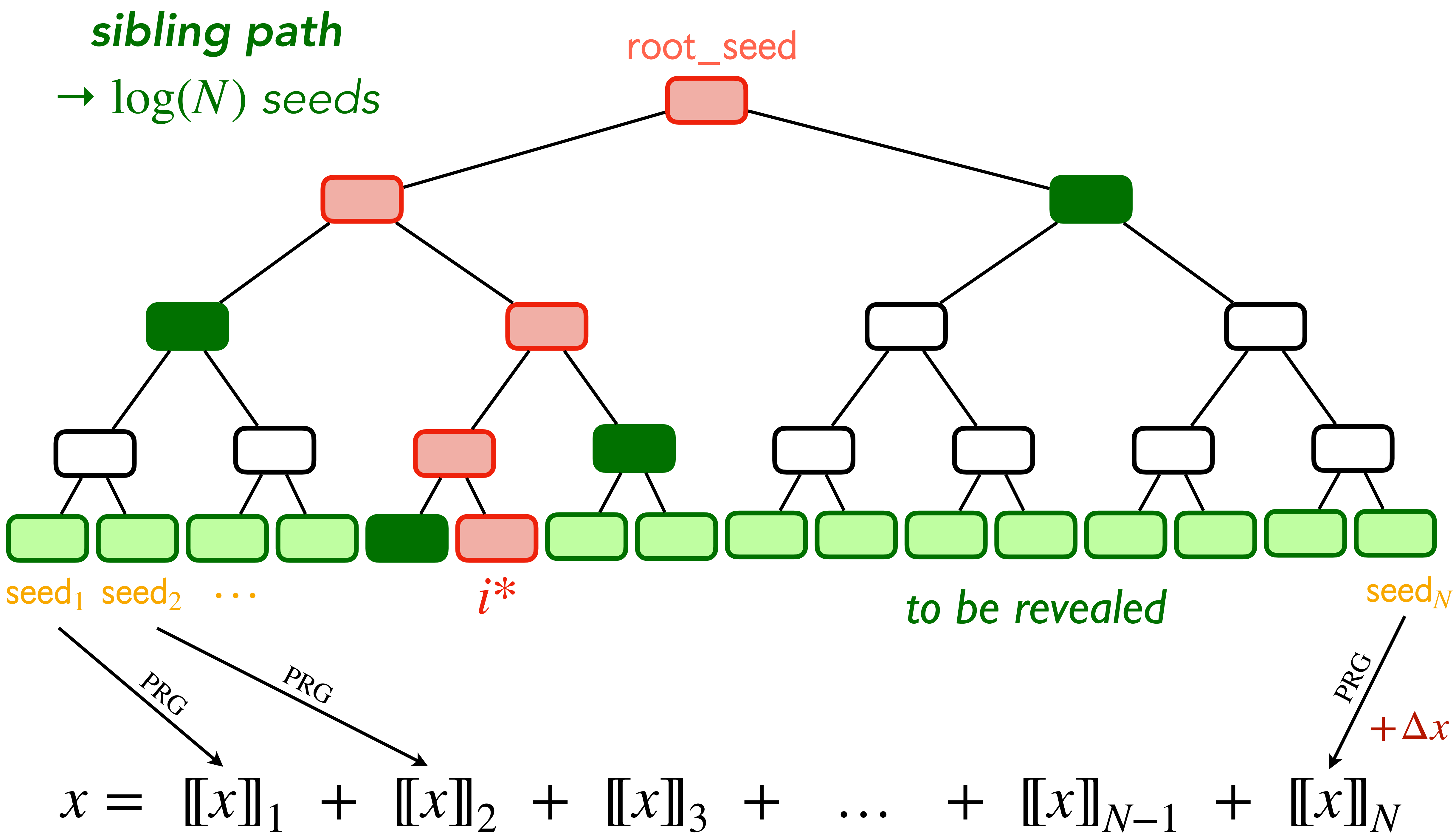

Traditional MPCitH transformation

Size of the broadcast

Size of the auxiliary value

Path in the seed (GGM) tree

$$\text{Size} \approx \tau \cdot \left(|\Delta x| + |\alpha| + \lambda \cdot \log_2 N + 2\lambda \right)$$

Commitment
of the hidden party

Number of repetitions to achieve the desired security level

$$\tau \approx \frac{\lambda}{\log_2 N}$$

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Commitment
of the hidden party

Number of repetitions to achieve the desired security level

$$\tau \approx \frac{\lambda}{\log_2 N}$$

Emulating τ MPC protocols with N parties is very expensive.

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

Traditional: one sharing of x

$$x = r_1 + r_2 + \ldots + r_N + \Delta x$$

The Hypercube Technique

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Traditional: one sharing of x

$$x = r_1 + r_2 + \ldots + r_N + \Delta x$$

Hypercube: D sharings of x, with the same auxiliary value Δx

$$x = \left\{ \begin{array}{c} r_{1,1} + r_{1,2} + \ldots + r_{1,N_1} \\ r_{2,1} + r_{2,2} + \ldots + r_{2,N_2} \\ \ldots \\ r_{D,1} + r_{D,2} + \ldots + r_{D,N_D} \end{array} \right\} + \Delta x$$

such that $N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$

The Hypercube Technique

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$$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$$

How to build these D sharings?

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

$$x = \begin{Bmatrix} r_{1,1} + r_{1,2} + \ldots + r_{1,N_1} \\ r_{2,1} + r_{2,2} + \ldots + r_{2,N_2} \\ \ldots \\ r_{D,1} + r_{D,2} + \ldots + r_{D,N_D} \end{Bmatrix} + \Delta x$$

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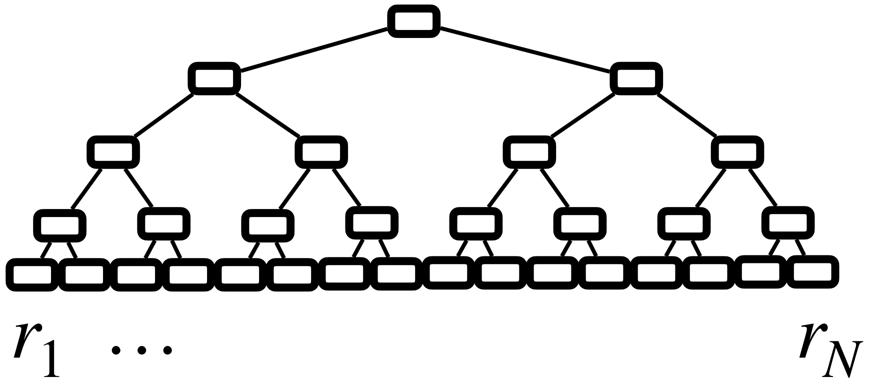

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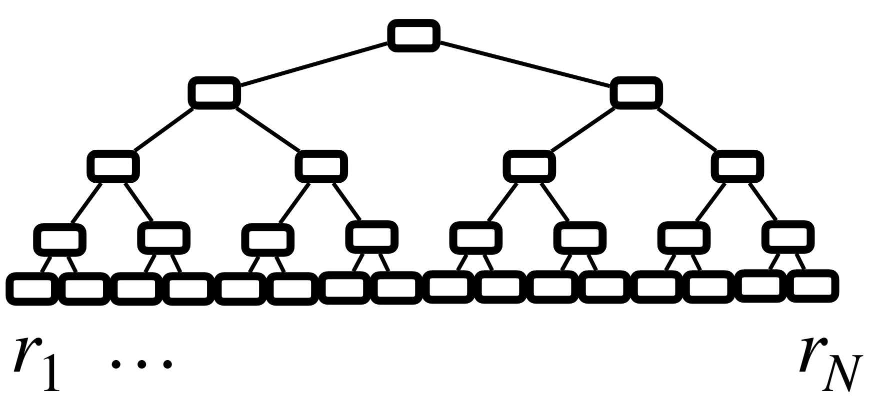
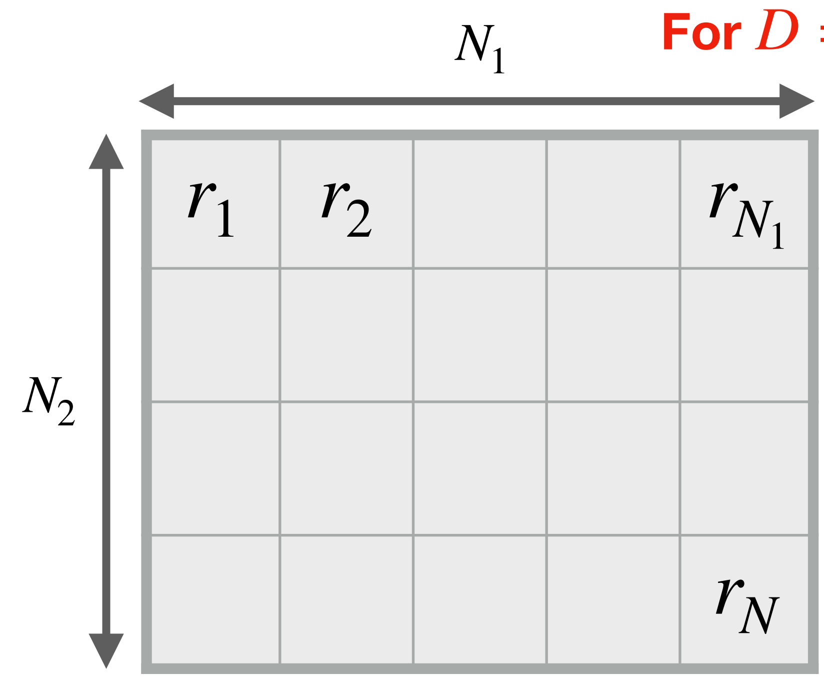
How to build these D sharings?



For $D = 2$



The Hypercube Technique

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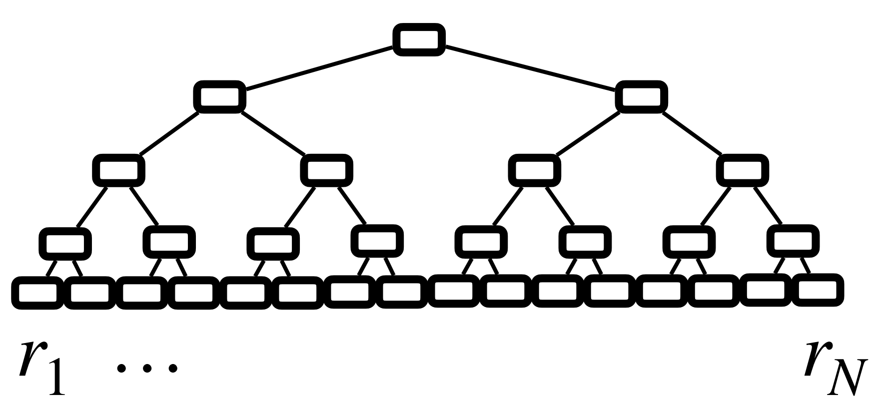
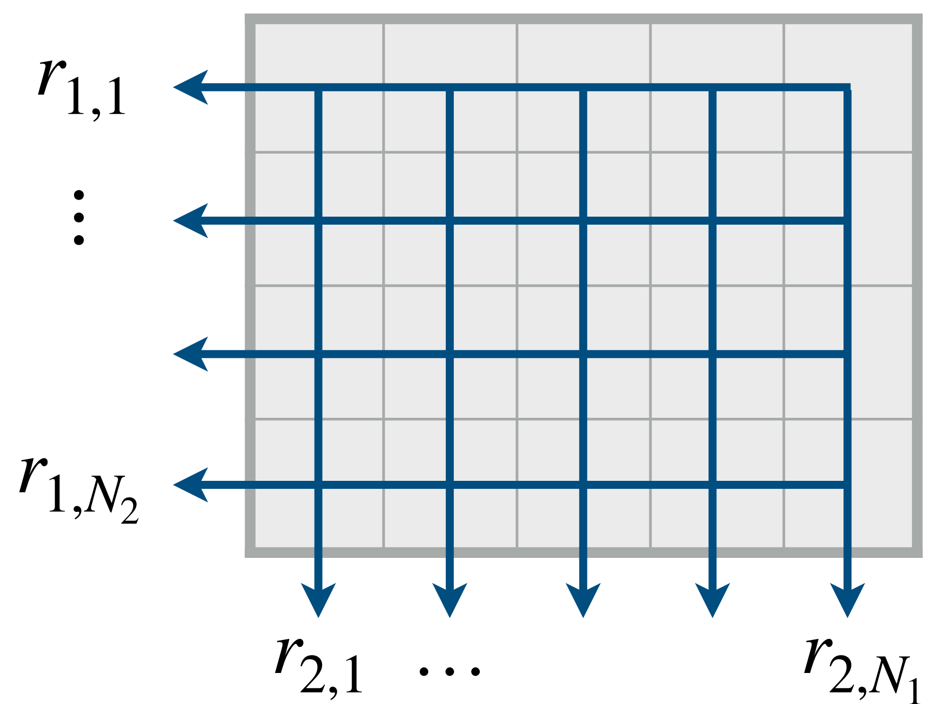
$$x = \left\{ \begin{matrix} r_{1,1} + r_{1,2} + \ldots + r_{1,N_1} \\ r_{2,1} + r_{2,2} + \ldots + r_{2,N_2} \\ \ldots \\ r_{D,1} + r_{D,2} + \ldots + r_{D,N_D} \end{matrix} \right\} + \Delta x$$

$$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$$

How to build these D sharings?

For $D = 2$





The Hypercube Technique

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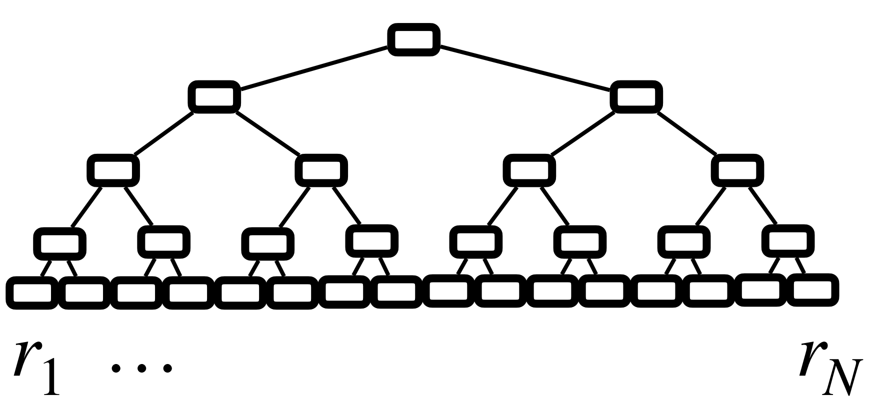
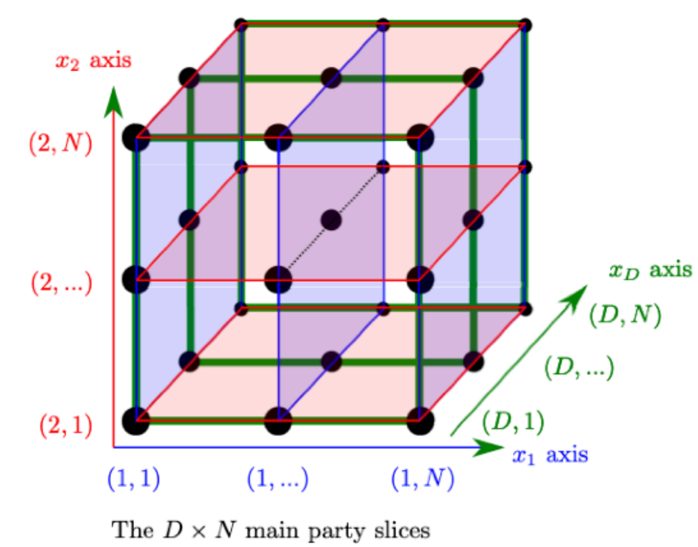
$$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$$

How to build these D sharings?

For $D \geq 2$





Source: Figure from [AGHHJY23]

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

$$x = \left\{ \begin{matrix} r_{1,1} + r_{1,2} + \ldots + r_{1,N_1} \\ r_{2,1} + r_{2,2} + \ldots + r_{2,N_2} \\ \ldots \\ r_{D,1} + r_{D,2} + \ldots + r_{D,N_D} \end{matrix} \right\} + \Delta x$$

$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$

What about the soundness?

We emulate D sub-protocols…

$$\frac{1}{N_1} \cdot \frac{1}{N_2} \cdot \ldots \cdot \frac{1}{N_D} = \frac{1}{N}$$

Same soundness error as before!

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

$$x = \begin{Bmatrix} r_{1,1} + r_{1,2} + \ldots + r_{1,N_1} \\ r_{2,1} + r_{2,2} + \ldots + r_{2,N_2} \\ \ldots \\ r_{D,1} + r_{D,2} + \ldots + r_{D,N_D} \end{Bmatrix} + \Delta x$$

$$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$$

What about the signature size?

$$\text{Size} \approx \tau \cdot \left(|\Delta x| + |\alpha| + \lambda \cdot \log_2 N + 2\lambda \right)$$

Same signature size as before!

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

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$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$

What about the emulation cost?

We emulate D sub-protocols…

$$N_1 + N_2 + \ldots + N_D$$

The Hypercube Technique

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The Hypercube Technique

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$$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$$

What about the emulation cost?

We emulate D sub-protocols…

$$\sout{N_1 + N_2 + \ldots + N_D}$$

$$1 + (N_1 - 1) + (N_2 - 1) + \ldots + (N_D - 1)$$

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

$$x = \left\{ \begin{array}{c} r_{1,1} + r_{1,2} + \ldots + r_{1,N_1} \\ r_{2,1} + r_{2,2} + \ldots + r_{2,N_2} \\ \ldots \\ r_{D,1} + r_{D,2} + \ldots + r_{D,N_D} \end{array} \right\} + \Delta x$$

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What about the emulation cost?

We emulate D sub-protocols…

$$\sout{N_1 + N_2 + \ldots + N_D}$$

$$1 + (N_1 - 1) + (N_2 - 1) + \ldots + (N_D - 1)$$

instead of $N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

$$x = \begin{Bmatrix} r_{1,1} + r_{1,2} + \ldots + r_{1,N_1} \\ r_{2,1} + r_{2,2} + \ldots + r_{2,N_2} \\ \ldots \\ r_{D,1} + r_{D,2} + \ldots + r_{D,N_D} \end{Bmatrix} + \Delta x$$

$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$

What about the emulation cost?

We emulate D sub-protocols…

~~$N_1 + N_2 + \ldots + N_D$~~

$$1 + (N_1 - 1) + (N_2 - 1) + \ldots + (N_D - 1)$$

$D = \log_2 N$
$N_1 = \ldots = N_D = 2$

$$1 + \log_2 N$$

instead of $N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

Traditional: N party emulations per repetition

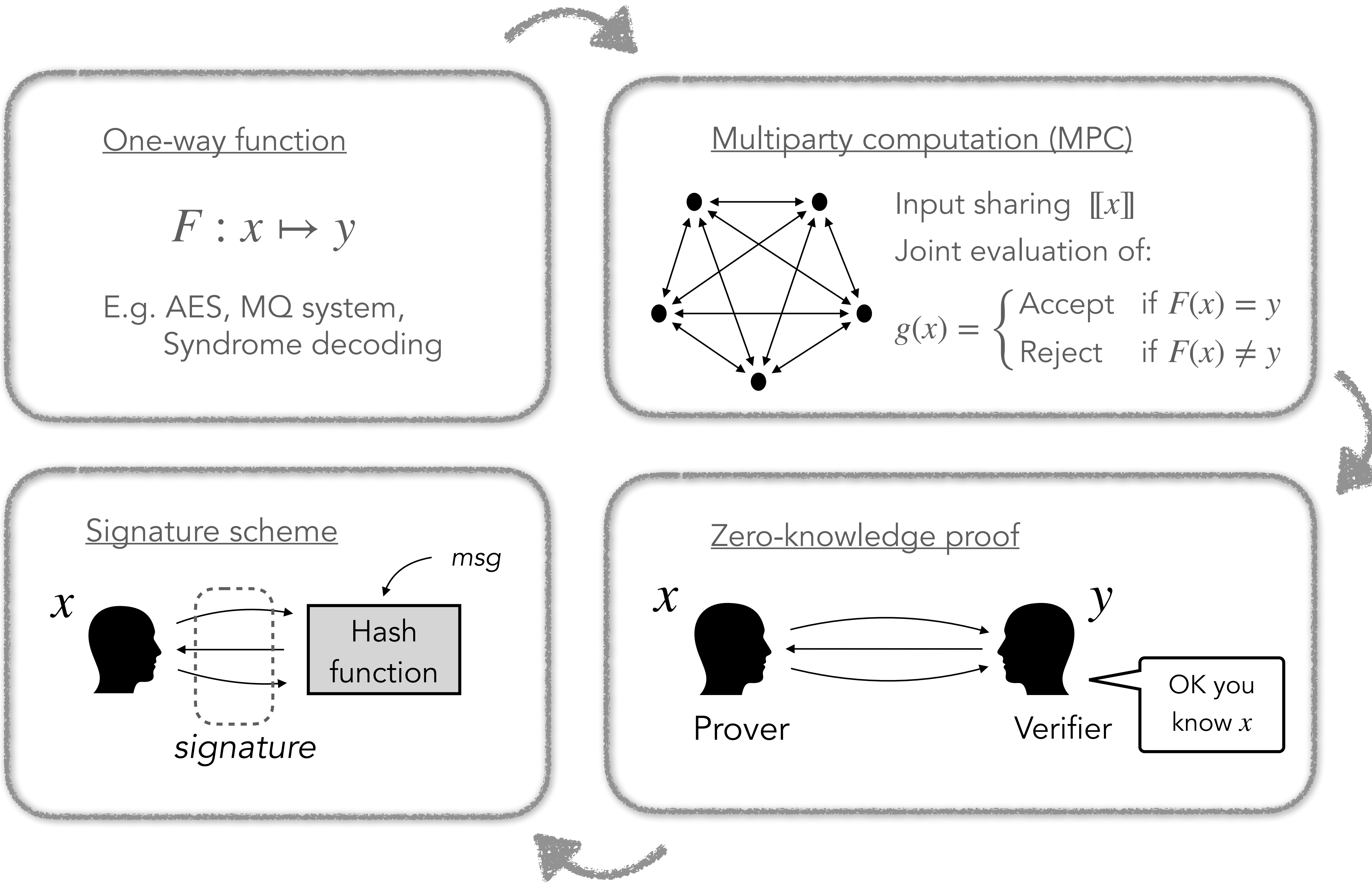
$$D = \log_2 N$$
$$N_1 = \ldots = N_D = 2$$

$$N = 256$$

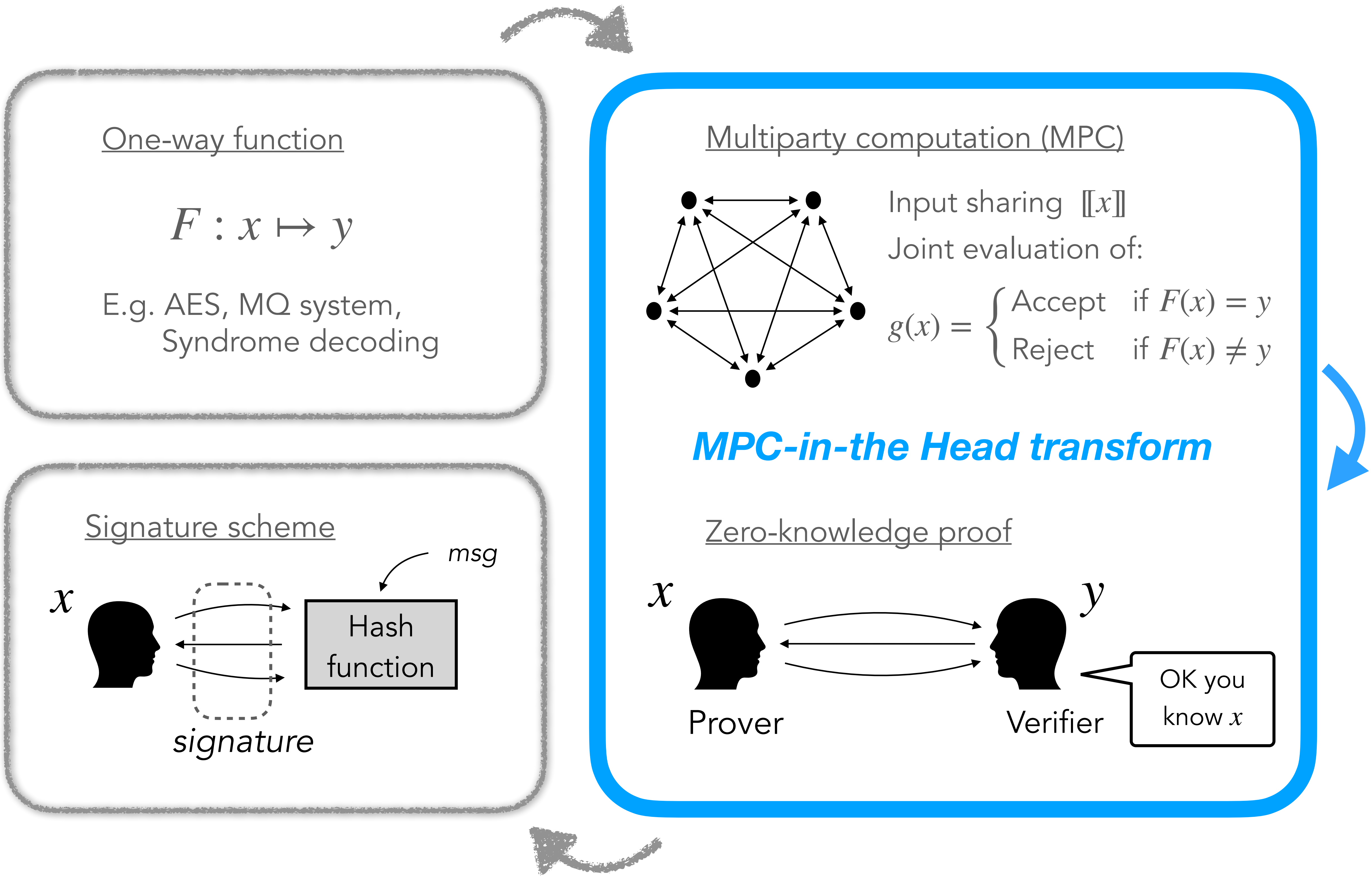
Hypercube: $1 + \log_2 N$ party emulations per repetition

$$1 + \log_2 N = 9$$

From MPC-in-the-Head to signatures



One-way function
$F : x \mapsto y$
E.g. AES, MQ system,
Syndrome decoding
Multiparty computation (MPC)
Input sharing $[\![x]\!]$
Joint evaluation of:
$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$
Signature scheme
msg
x
Hash function
signature
Zero-knowledge proof
x
y
Prover
Verifier
OK you know x



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F : x ↦ y
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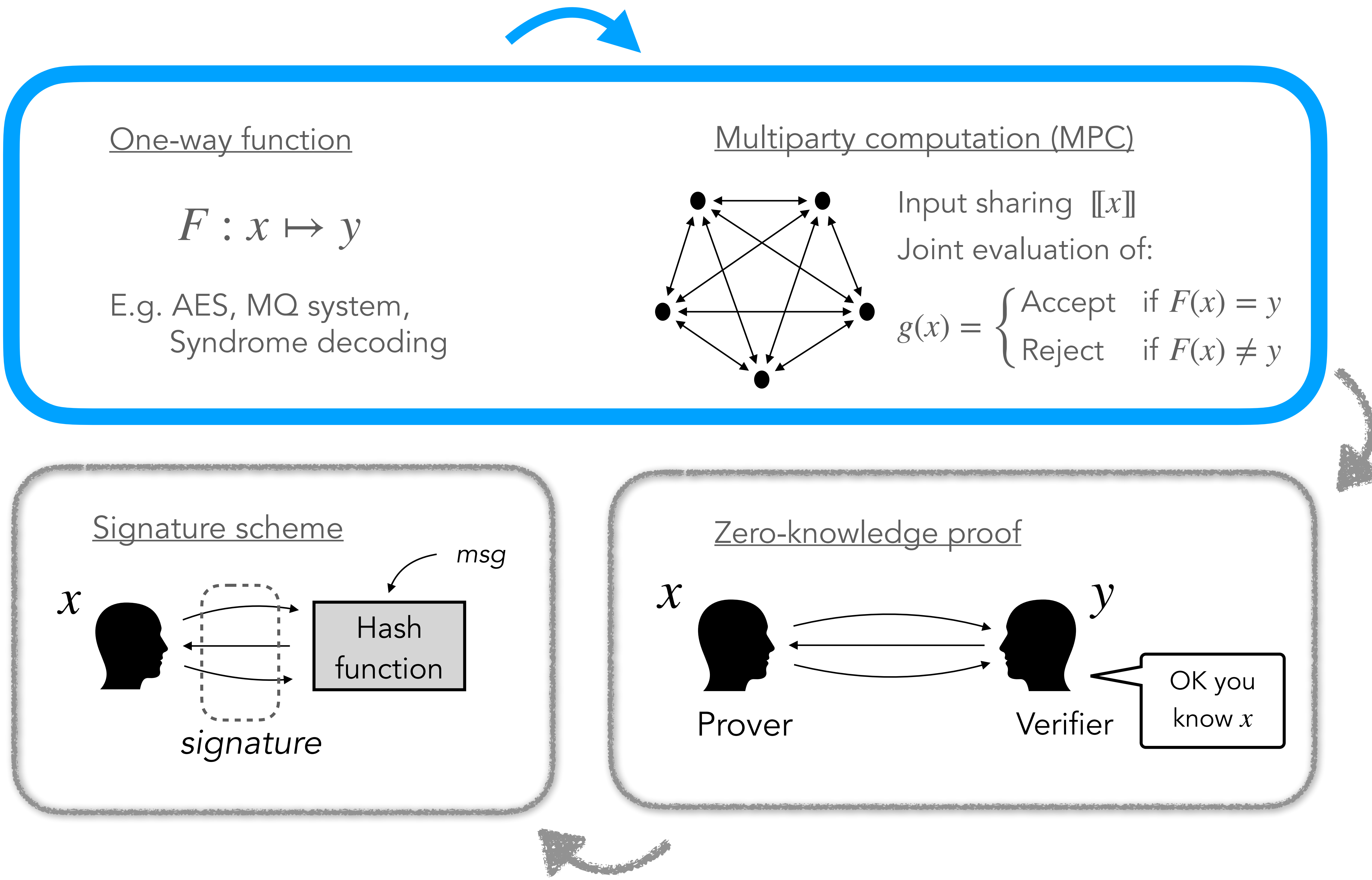
Syndrome decoding
Multiparty computation (MPC)
Input sharing ⟦x⟧
Joint evaluation of:
g(x) = { Accept if F(x) = y ; Reject if F(x) ≠ y
MPC-in-the Head transform
Zero-knowledge proof
x
y
Prover
Verifier
OK you know x
Signature scheme
x
msg
Hash function
signature



One-way function
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Input sharing $[\![x]\!]$
Joint evaluation of:
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Signature scheme
x
msg
Hash function
signature
Zero-knowledge proof
x
y
Prover
Verifier
OK you know x

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

- RYDE: Syndrome decoding problem in the rank metric

  From a matrix $H \in \mathbb{F}_{q^m}^{(n-k)\times n}$ and a vector $y \in \mathbb{F}_{q^m}^{n-k}$, find a vector $x \in \mathbb{F}_{q^m}^{n}$ s.t.

$$y = Hx \quad \text{and} \quad \dim \operatorname{Span}_{\mathbb{F}_q}(x_1, \ldots, x_n) \leq r.$$

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

- RYDE: Syndrome decoding problem in the rank metric

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- MIRA: MinRank problem

  From $k+1$ matrices $M_0, \ldots, M_k \in \mathbb{F}_q^{m\times n}$, find a vector $x \in \mathbb{F}_q^k$ such that

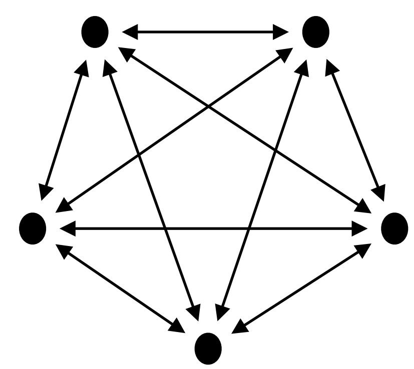
$$\mathrm{rank}\left(M_0 + \sum_{i=1}^{k} x_i M_i\right) \leq r.$$

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

Multiparty computation (MPC)



Input sharing $[\![x]\!]$

Joint evaluation of:

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

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Rank Syndrome Decoding Problem

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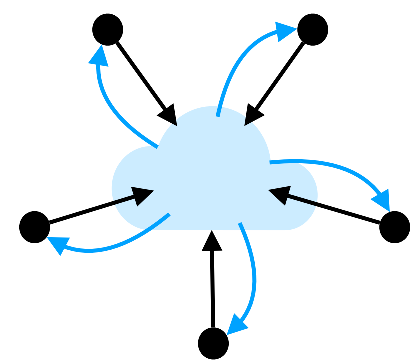
Multiparty computation (MPC)



Input sharing $[\![x]\!]$

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The case of ***RYDE***

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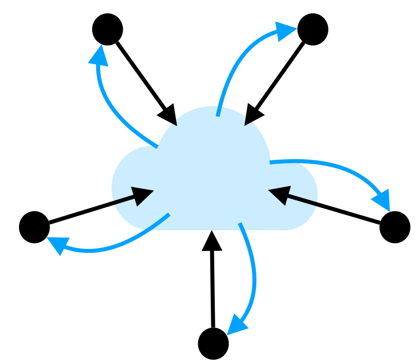

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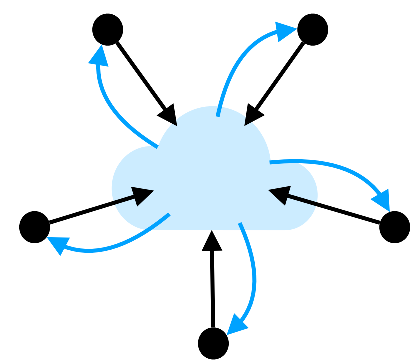

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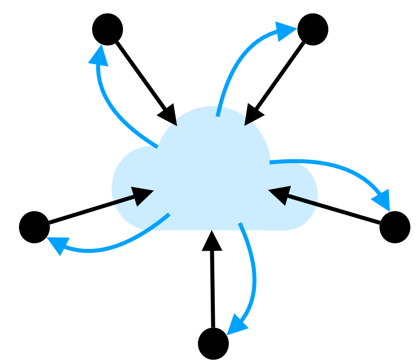

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The case of RYDE

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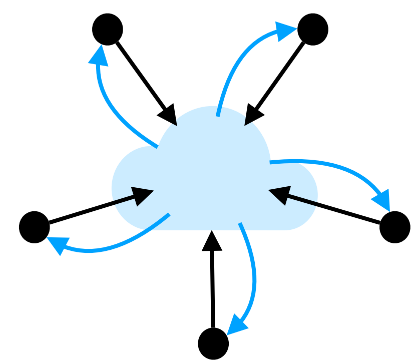

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[Fen22] Feneuil: "Building MPCitH-based Signatures from MQ, MinRank, Rank SD and PKP" (ePrint 2022/1512)

$$\dim \mathrm{Span}_{\mathbb{F}_q}(x_1, \ldots, x_n) \leq r$$

$$\Longleftrightarrow$$

$$\exists \text{ degree-}q^r\ q\text{-polynomial } P := \sum_{i=0}^{r} a_i X^{q^i} \text{ s.t. } P(x_1) = \ldots = P(x_n) = 0$$

The case of ***RYDE***

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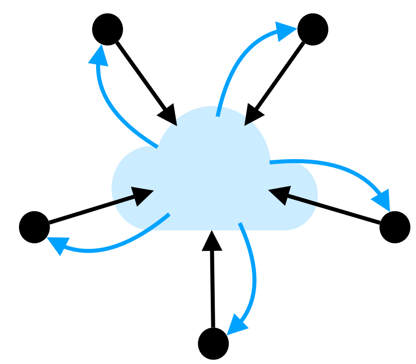

Input sharings $[\![x]\!]$ and $[\![P]\!]$

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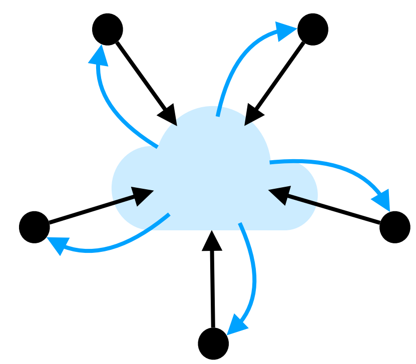

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- The parties will compute $[\![x_j^{q^i}]\!]$ from $[\![x_j]\!]$ for all (i, j).

The case of RYDE

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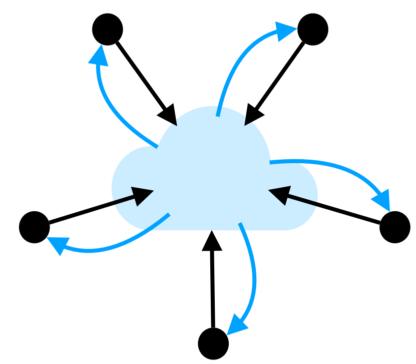

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- *Input of the MPC protocol*: two sharings $[\![x]\!]$ and $[\![P]\!] := \sum_{i=0}^{r} [\![a_i]\!] X^{q^i}$.
- The parties will compute $[\![x_j^{q^i}]\!]$ from $[\![x_j]\!]$ for all (i, j).
- For all j, the parties will check that $P(x_j) = 0$ by checking that

$$\left\langle \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_r \end{pmatrix}, \begin{pmatrix} x_j \\ x_j^q \\ \vdots \\ x_j^{q^r} \end{pmatrix} \right\rangle = 0,$$

using a [BN20]-like MPC protocol.

[BN20] Baum, Nof: "Concretely-efficient zero-knowledge arguments for arithmetic circuits and their application to lattice-based cryptography" (PKC 2020)

The case of RYDE

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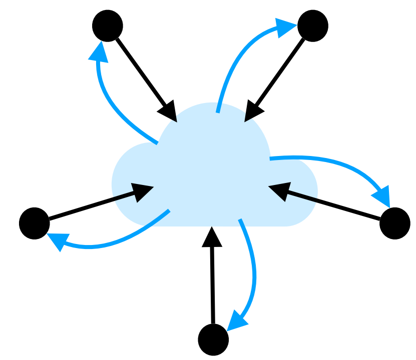

Input sharings $[\![x]\!]$ and $[\![P]\!]$

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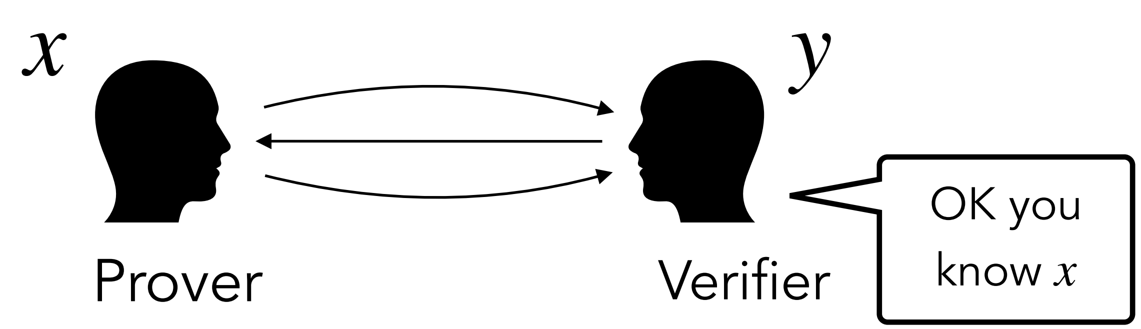
- Check the rank constraint

Zero-knowledge proof



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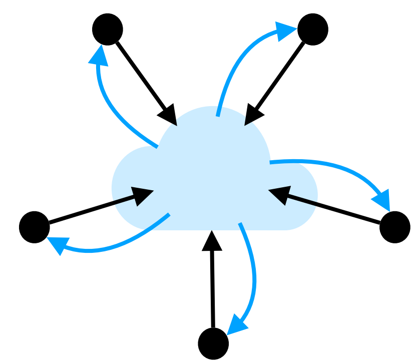

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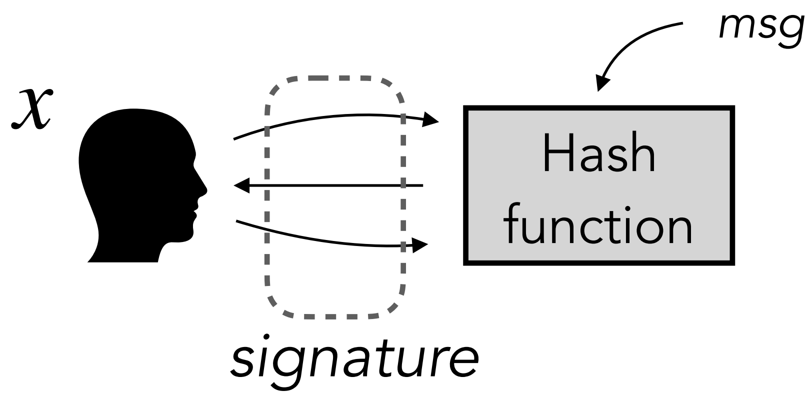
RYDE



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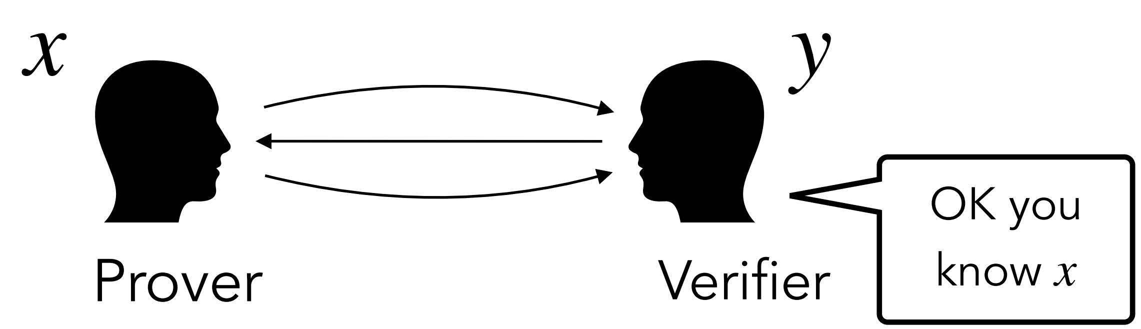

Fiat-Shamir transform

Should take [KZ20] attack into account (since there are 5 rounds)!

[KZ20] Kales, Zaverucha. "An attack on some signature schemes constructed from five-pass identification schemes" (CANS20)

MinRank Problem

From $M_0, \ldots, M_k \in \mathbb{F}_q^{m \times n}$, find x such that

$$\operatorname{rank}\left(M_0 + \sum_{i=1}^{k} x_i M_i\right) \leq r.$$

The case of MIRA

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From $M_0, \ldots, M_k \in \mathbb{F}_q^{m \times n}$, find x such that

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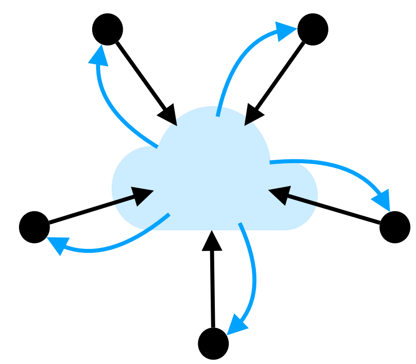
Multiparty computation (MPC)



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- Let us denote $E = M_0 + \sum_{i=1}^{k} x_i M_i \in \mathbb{F}_q^{m \times n}$.

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From $M_0, \ldots, M_k \in \mathbb{F}_q^{m \times n}$, find x such that

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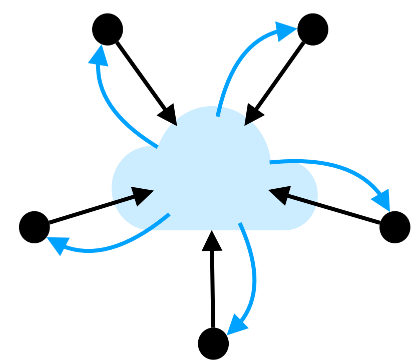

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$$\text{rank}(E) \leq r \iff \dim \text{Span}_{\mathbb{F}_q}(e_1, \ldots, e_n) \leq r.$$

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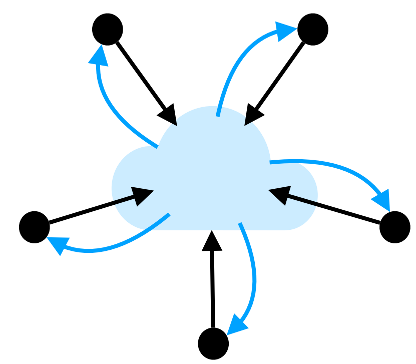

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$$\operatorname{rank}(E) \le r \iff \dim \operatorname{Span}_{\mathbb{F}_q}(e_1, \ldots, e_n) \le r.$$
- We can use the *same protocol* than with RYDE.

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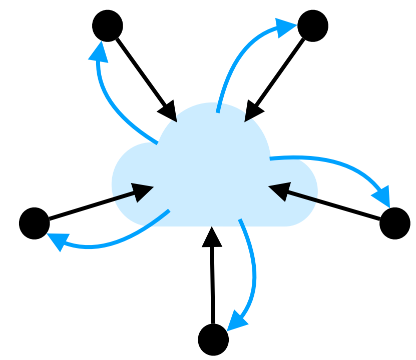

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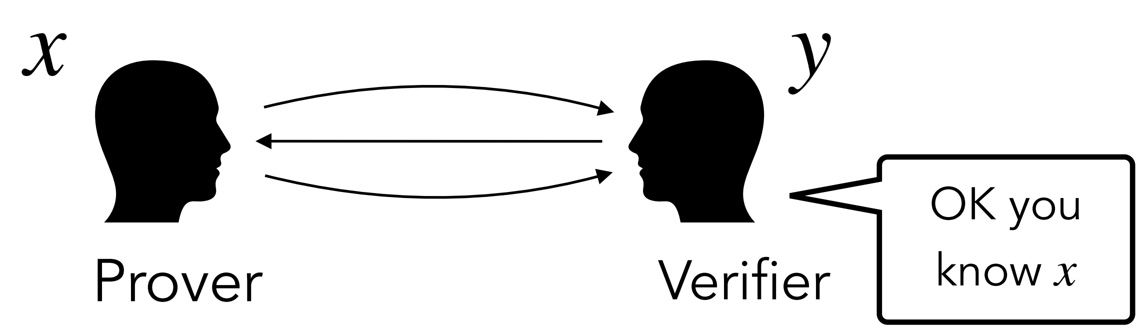

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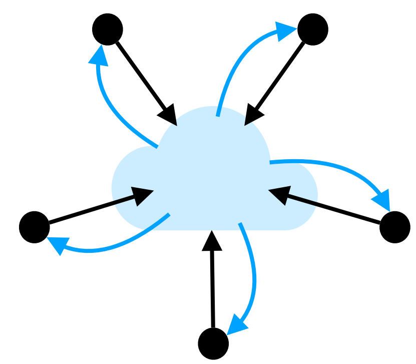

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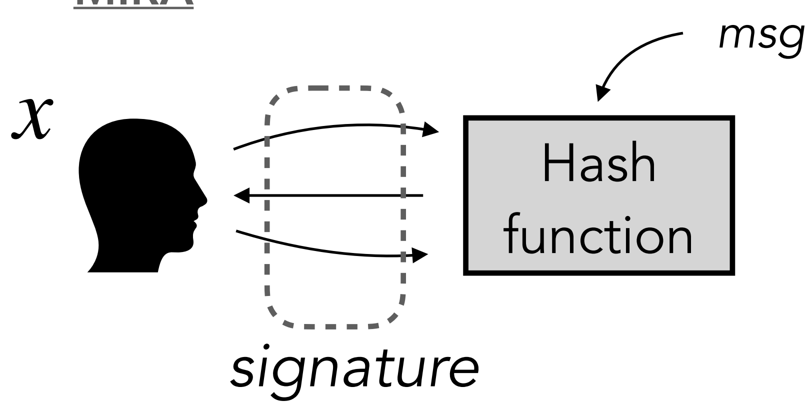

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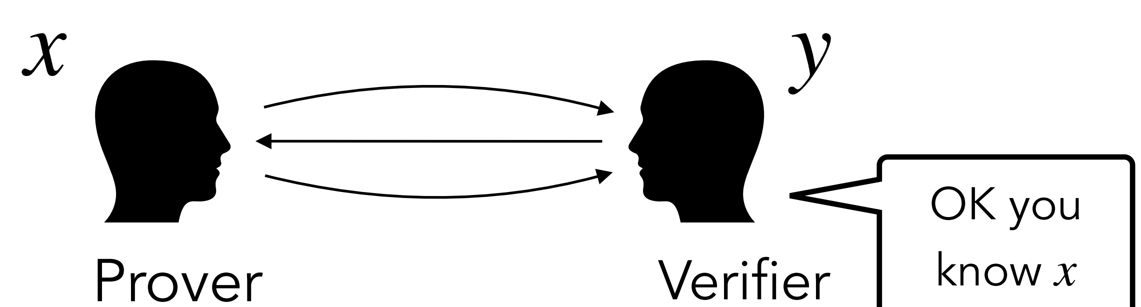

Performances

Performances

	Short Instance			Fast Instance		
	$\|sig\|$	t_{sign}	t_{verify}	$\|sig\|$	t_{sign}	t_{verify}
RYDE L1	6.0	23.4	20.1	7.4	5.4	4.4
MIRA L1	5.6	46.8	43.9	7.4	37.4	36.7
RYDE L3	12.9	49.6	44.8	16.4	12.2	10.7
MIRA L3	11.8	119.7	116.2	15.5	107.2	107.0
RYDE L5	22.8	105.5	94.9	29.1	26.0	22.7
MIRA L5	20.8	337.7	331.4	27.7	322.3	323.2

All public keys are smaller than 200 bytes.

Isochronous implementations
Size in kilobytes, timing in Mcycles
@2.60GHz: 1 millisecond ≈ 2.6 Mcycles

Advantages and limitations

- <u>Limitations</u>
  - Relatively ***slow*** *(few milliseconds)*
    - Greedy use of symmetric cryptography
  - Relatively ***large*** signatures (*5.5-7.5 KB for L1*)
  - Signature size: ***quadratic*** growth in the security level

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- Advantages
  - ***Conservative*** hardness assumption:
    - No structure, no trapdoor
  - ***Small*** (public) keys
  - ***Good*** public key + signature size
  - Adaptive and ***tunable*** parameters

Rank Metric in the Head

RYDE

N. Aragon, M. Bardet, L. Bidoux, J.-J. Chi-Domínguez, V. Dyseryn, T. Feneuil, P. Gaborit, A. Joux, M. Rivain, J.-P. Tillich, A. Vincotte

`https://pqc-ryde.org`

MIRA

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`https://pqc-mira.org`

Thank you for your attention.