Post-Quantum Signatures from Multiparty Computation: Recent Advances

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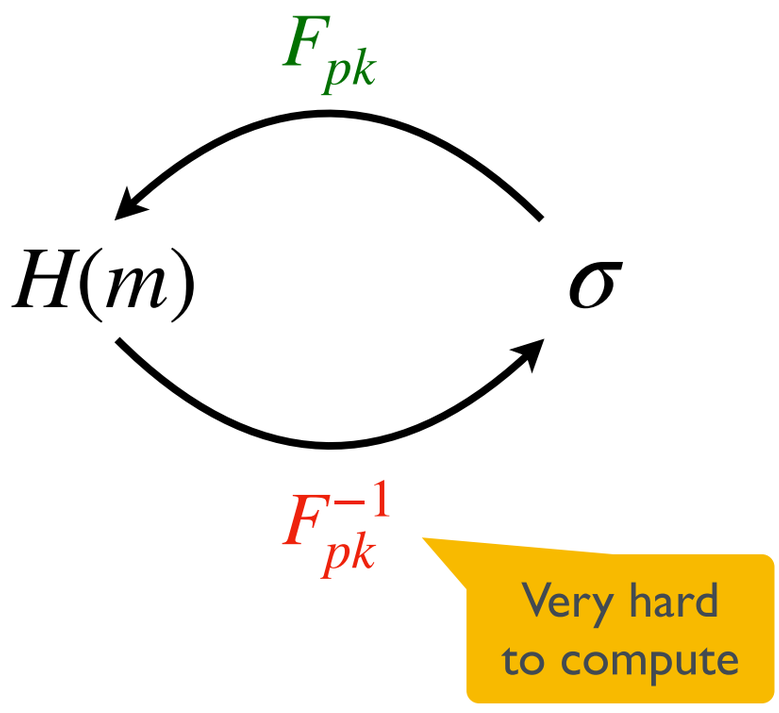
Some figures used in the following slides have been realised by Matthieu Rivain (CryptoExperts).

Introduction

How to build signature schemes?

Hash & Sign



- Short signatures
- “Trapdoor” in the public key

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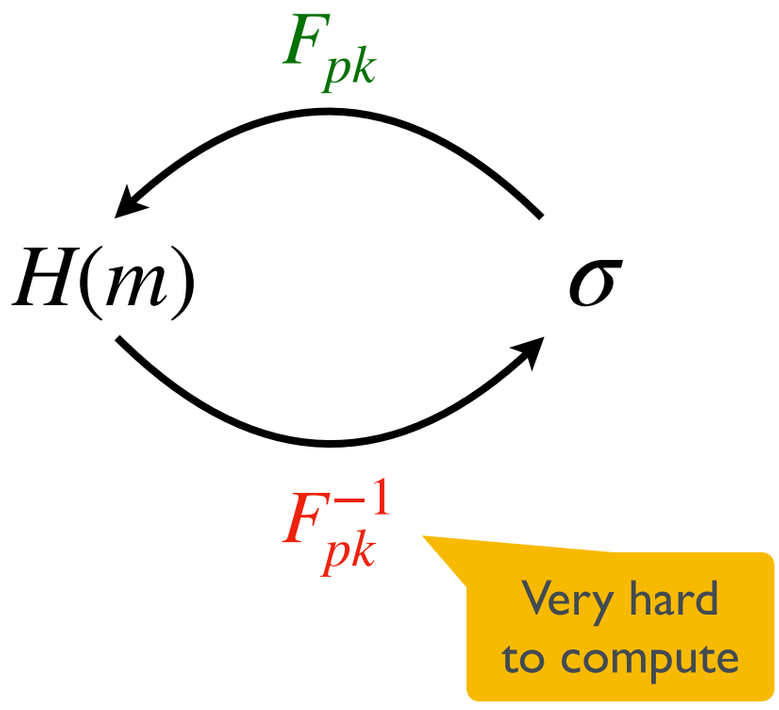

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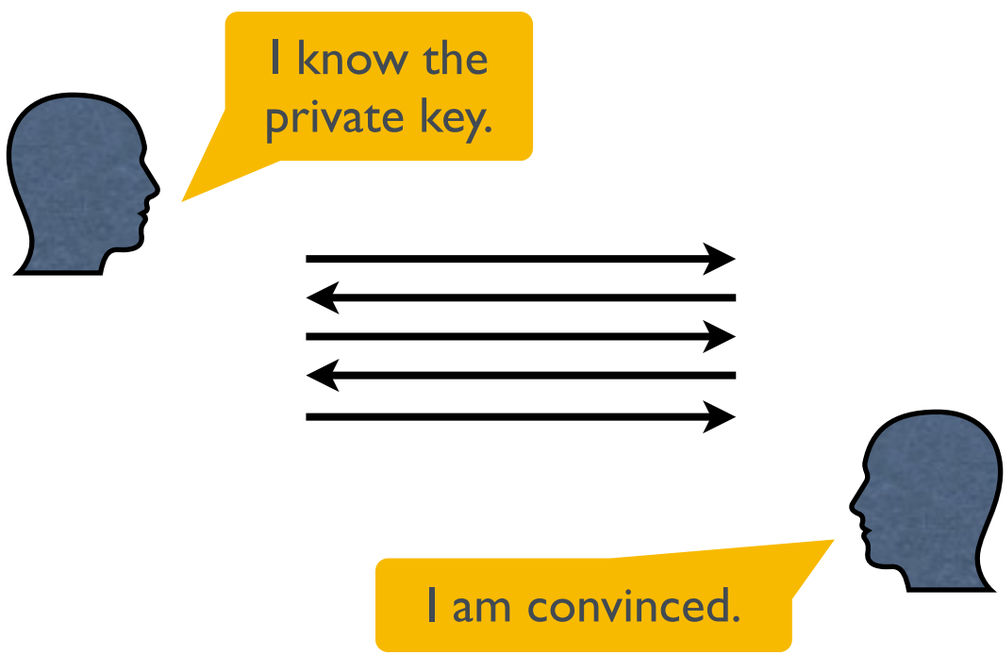
From a zero-knowledge proof



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- Short public key

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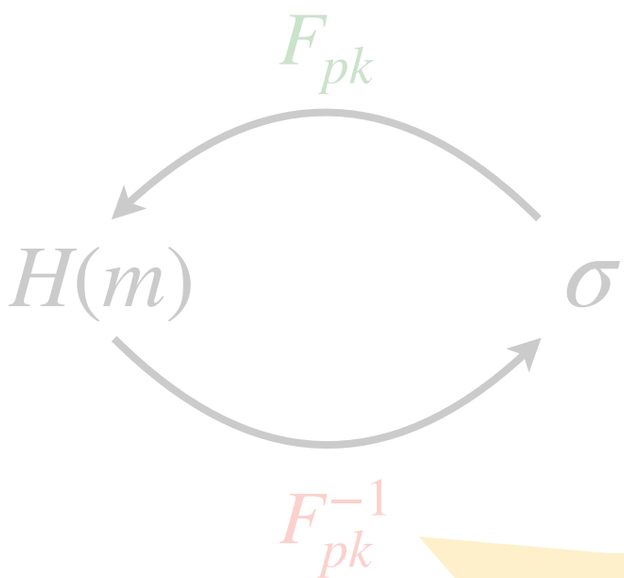

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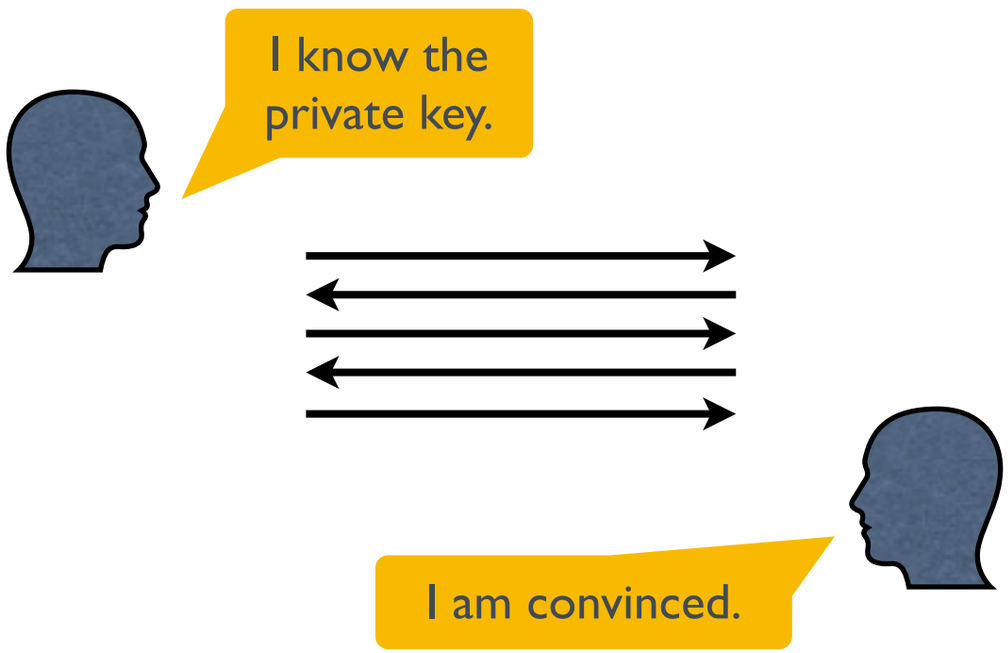

- Large(r) signatures
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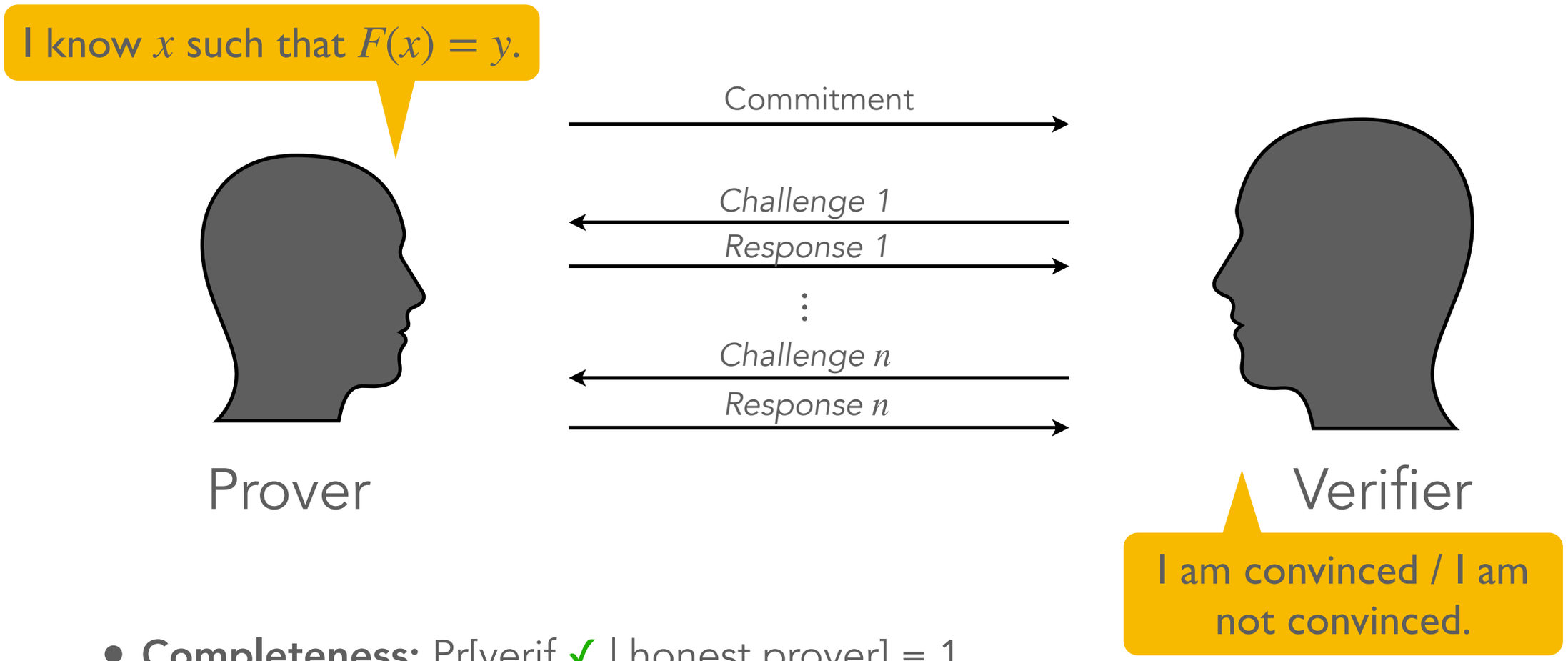
Proof of knowledge



- **Completeness:** Pr[verif ✓ | honest prover] = 1
- **Soundness:** Pr[verif ✓ | malicious prover] $\leq \varepsilon$ (*e.g.* 2^{-128})
- **Zero-knowledge:** verifier learns nothing on x

MPC in the Head

- **[IKOS07]** Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, Amit Sahai: "Zero-knowledge from secure multiparty computation" (STOC 2007)
- Turn an MPC protocol into a zero knowledge proof of knowledge
- **Generic**: can be apply to any cryptographic problem

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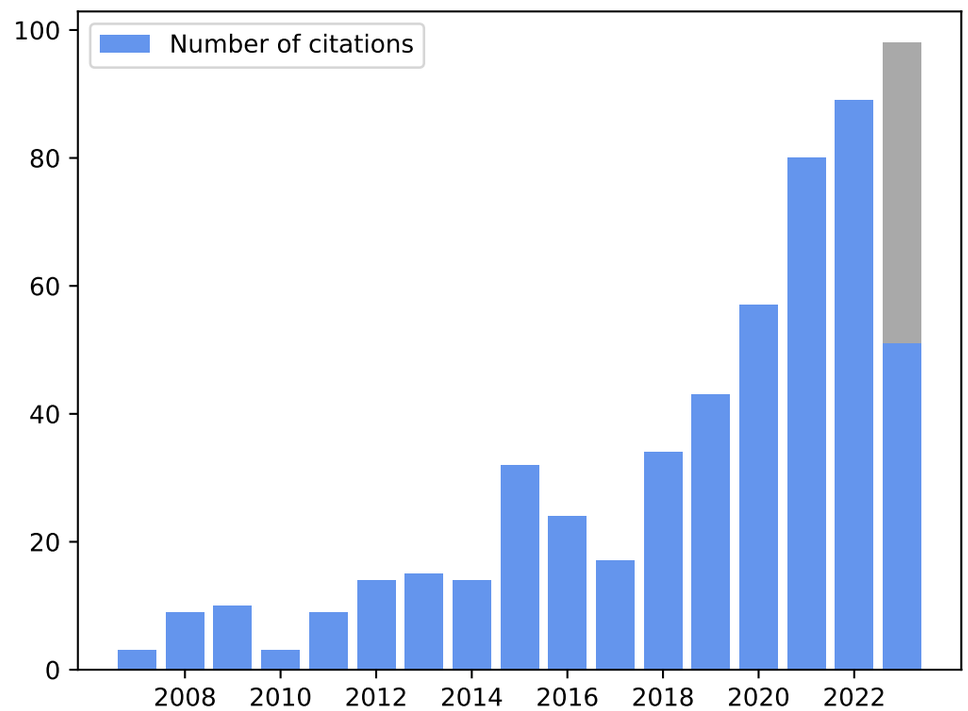

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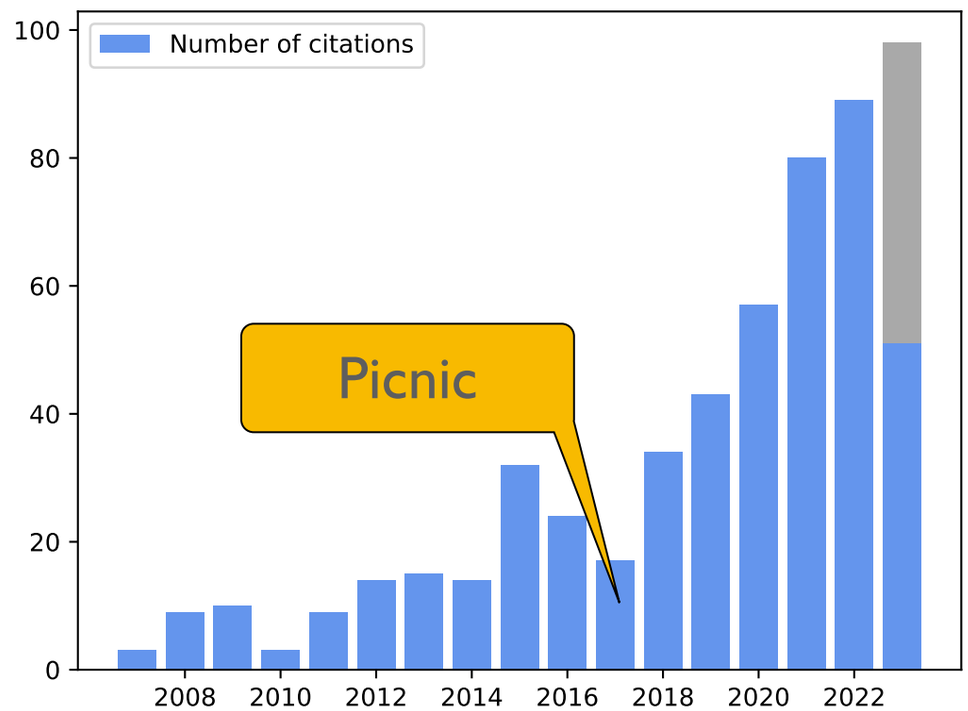

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- Turn an MPC protocol into a zero knowledge proof of knowledge
- **Generic**: can be apply to any cryptographic problem
- Convenient to build (candidate) **post-quantum signature** schemes
- **Picnic**: submission to NIST (2017)
- First round of recent NIST call: 8 MPCitH schemes / 40 submissions

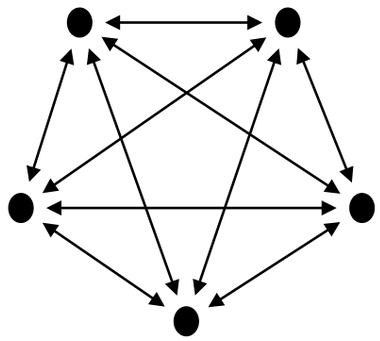
AIMer	*MQOM*
Biscuit	*PERK*
MIRA	*RYDE*
MiRitH	*SDitH*

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

Multiparty computation (MPC)

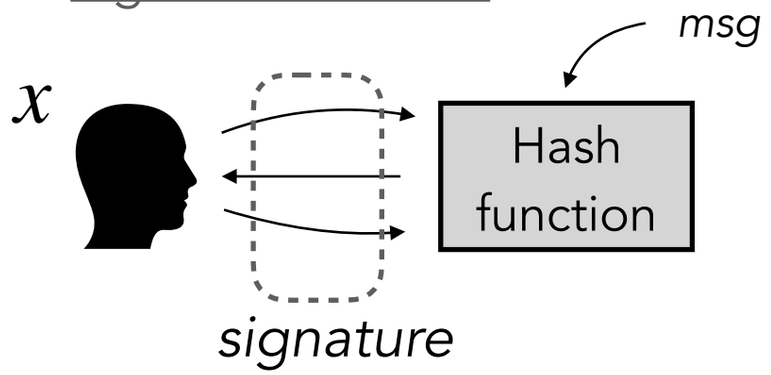


Input sharing $[\![x]\!]$

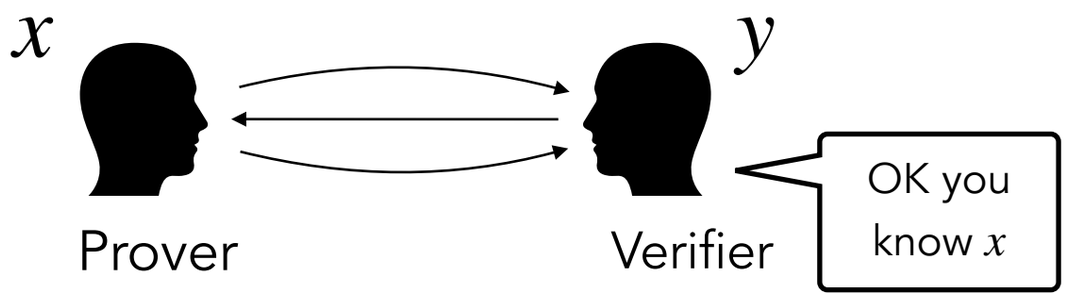
Joint evaluation of:

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

Signature scheme



Zero-knowledge proof



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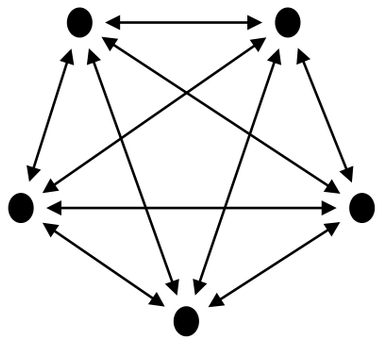

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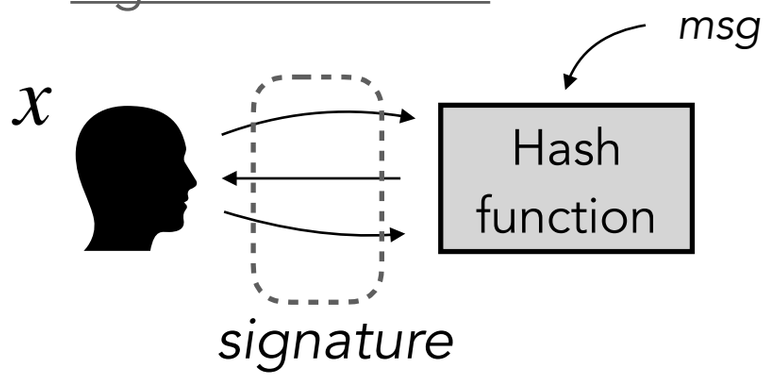

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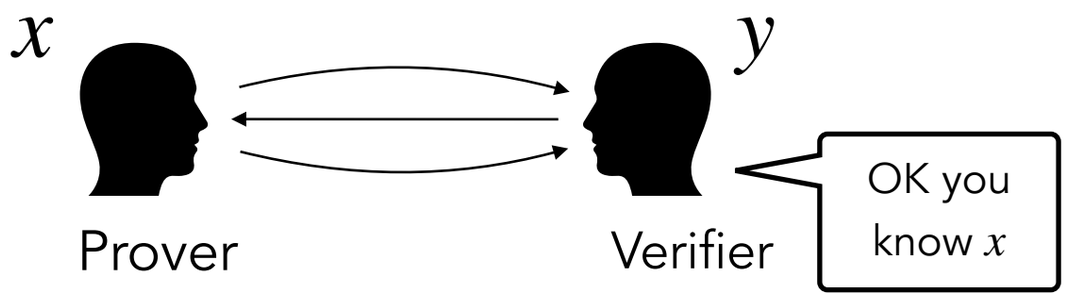

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$ s.t. $x = [\![x]\!]_1 + \ldots + [\![x]\!]_N$

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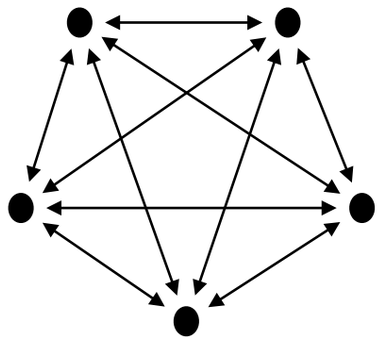

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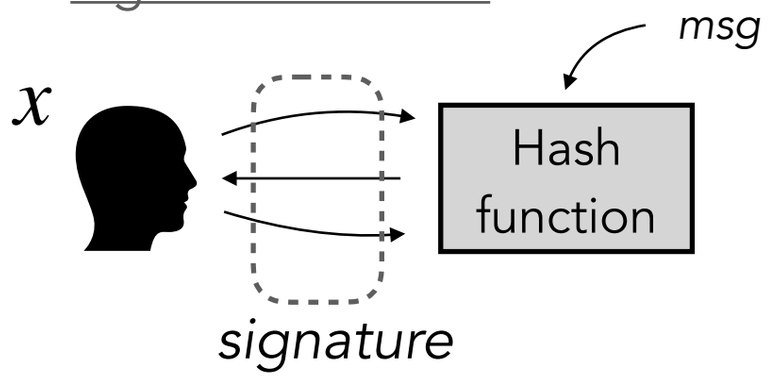

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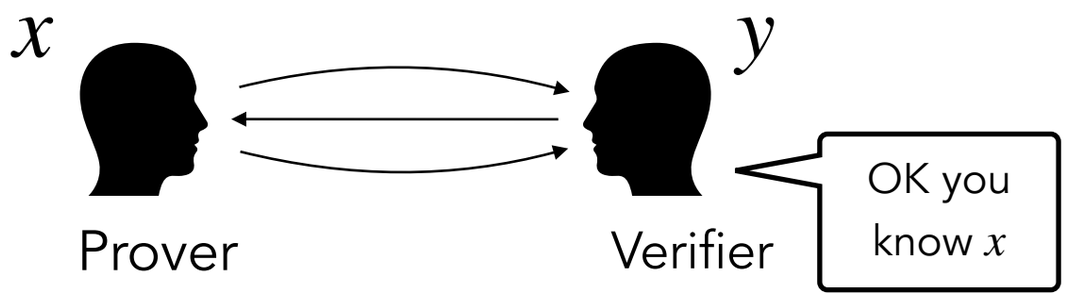

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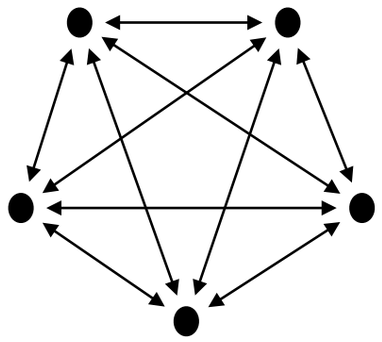

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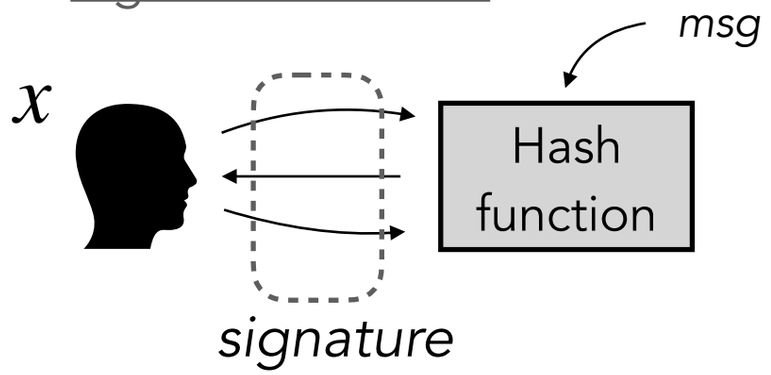

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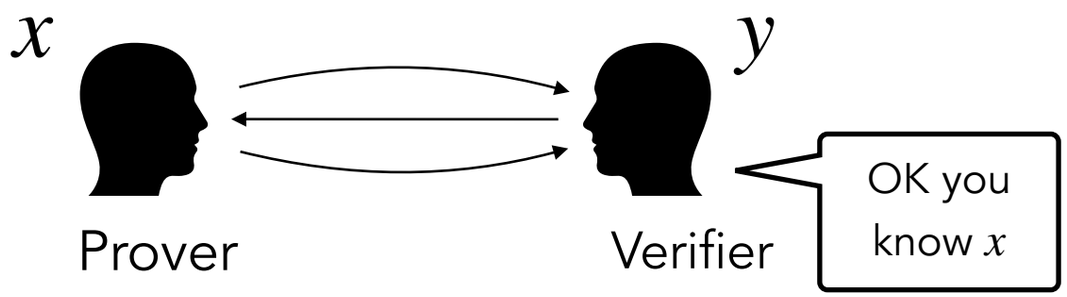

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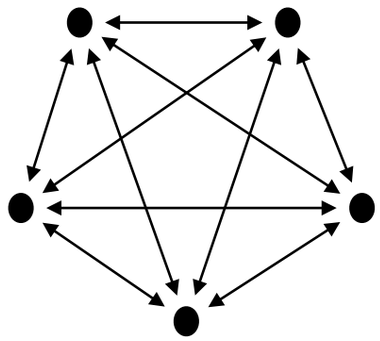

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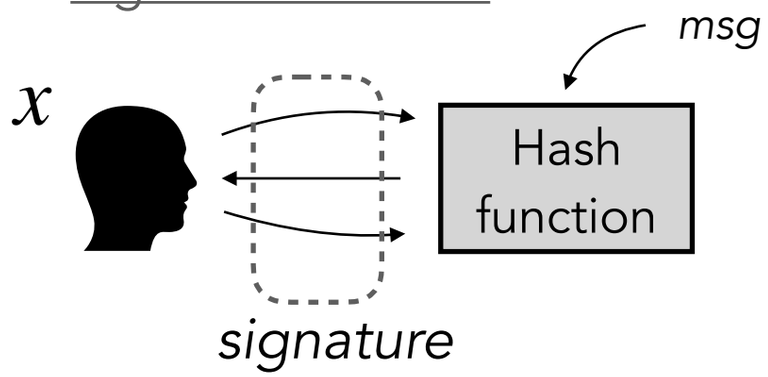

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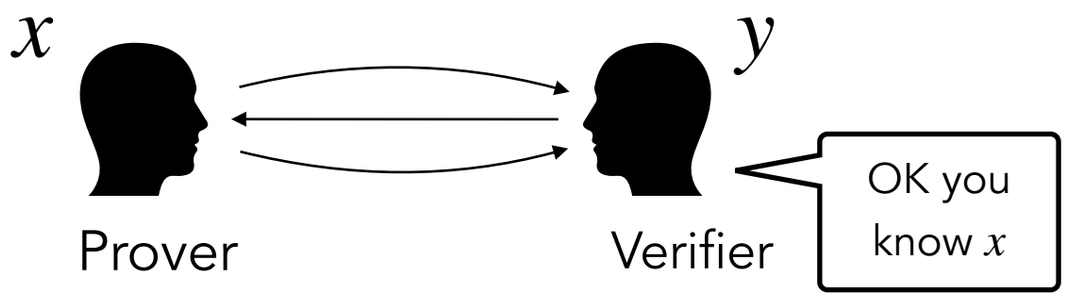

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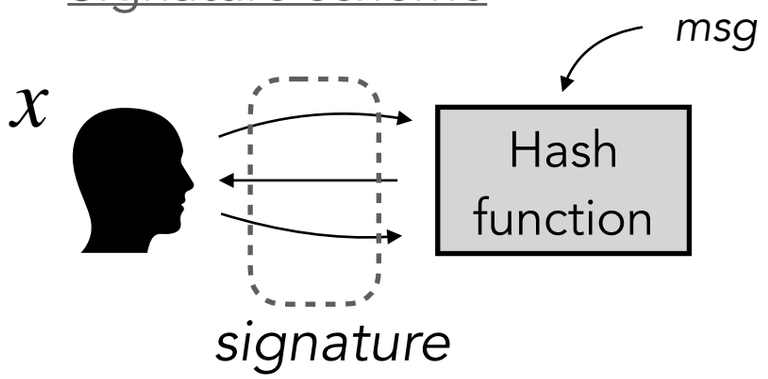

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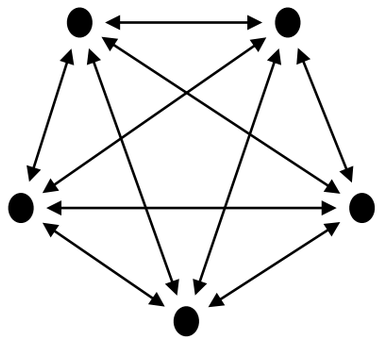

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MPC-in-the-Head transform

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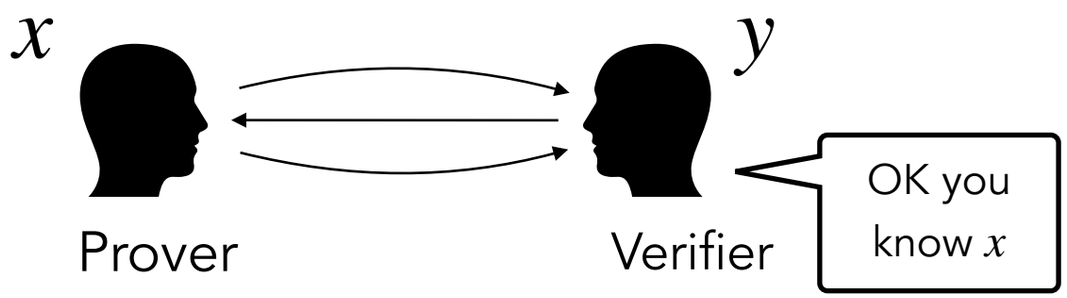
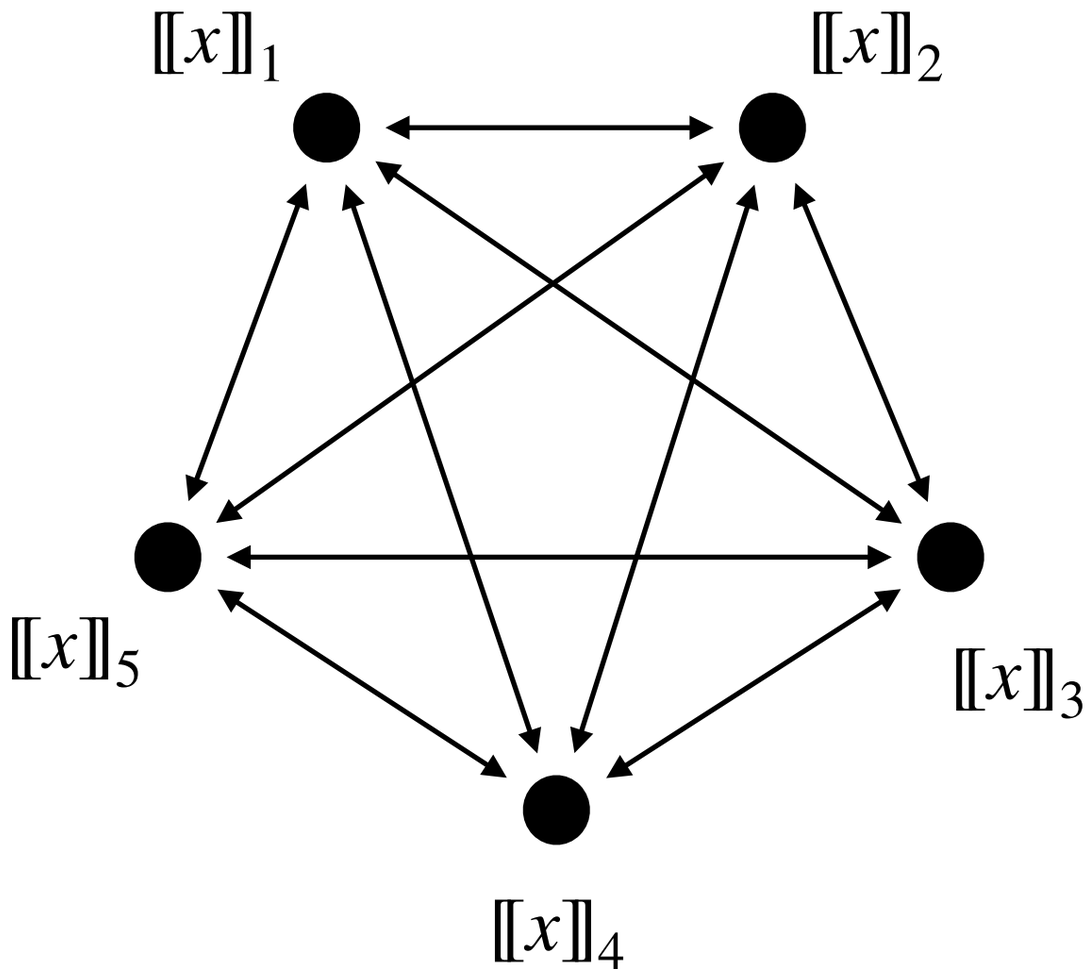


MPCitH: general principle

MPC model



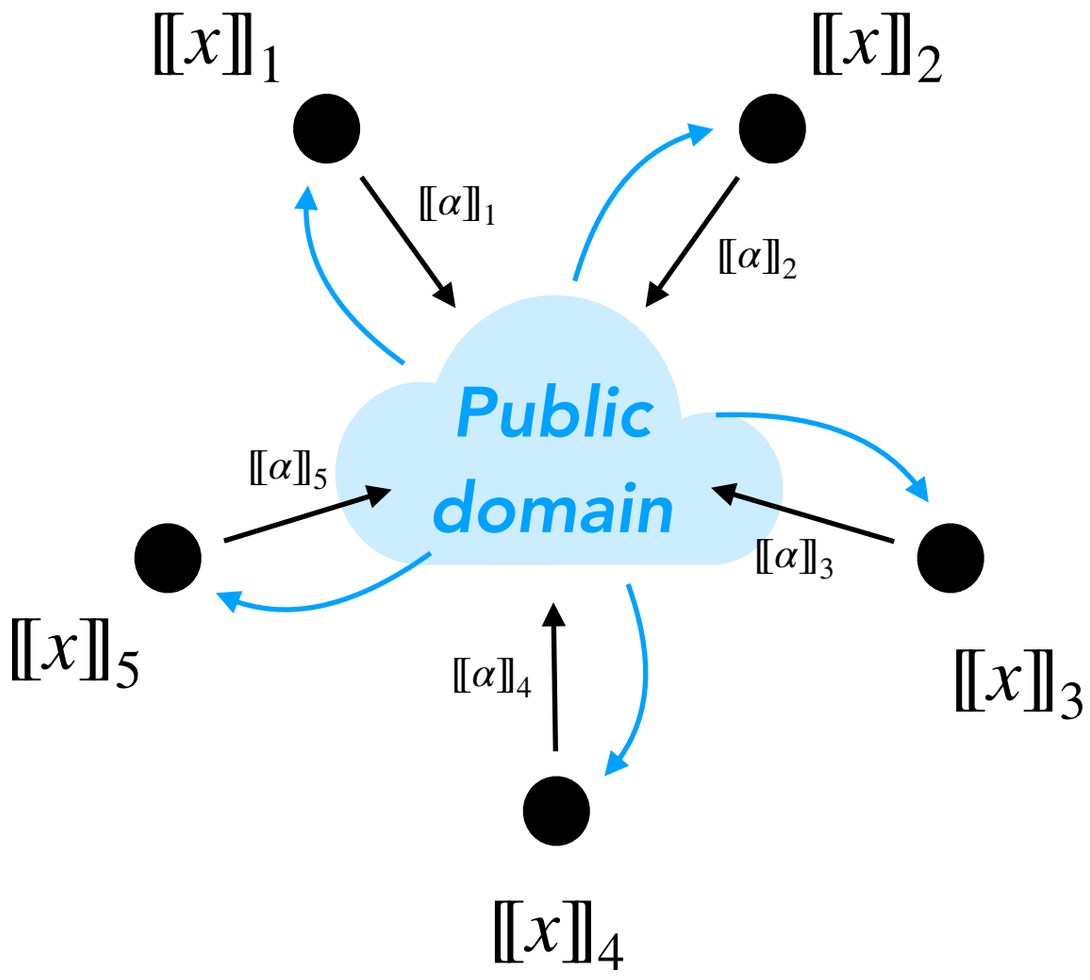
$$x = [\![x]\!]_1 + [\![x]\!]_2 + \ldots + [\![x]\!]_N$$

- **Jointly compute**

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

- $(N-1)$ **private:** the views of any $N-1$ parties provide no information on x
- **Semi-honest model:** assuming that the parties follow the steps of the protocol

MPC model



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- $(N-1)$ **private:** the views of any $N-1$ parties provide no information on x
- **Semi-honest model:** assuming that the parties follow the steps of the protocol
- ***Broadcast model***
  - *Parties locally compute on their shares* $[\![x]\!] \mapsto [\![\alpha]\!]$
  - *Parties broadcast* $[\![\alpha]\!]$ *and recompute* α
  - *Parties start again (now knowing* α*)*

MPCitH transform

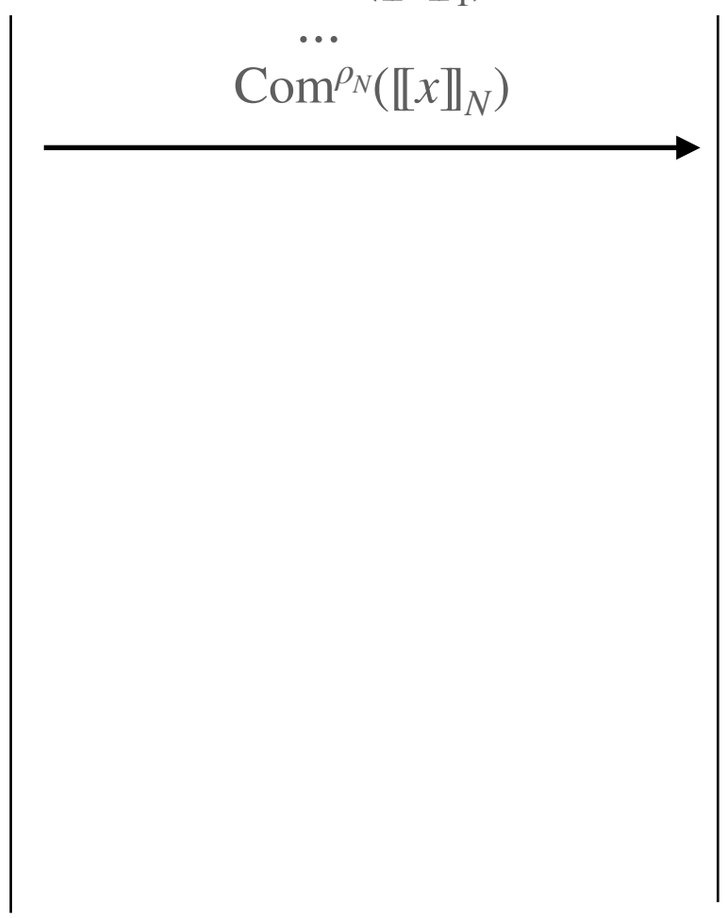
Prover

Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$



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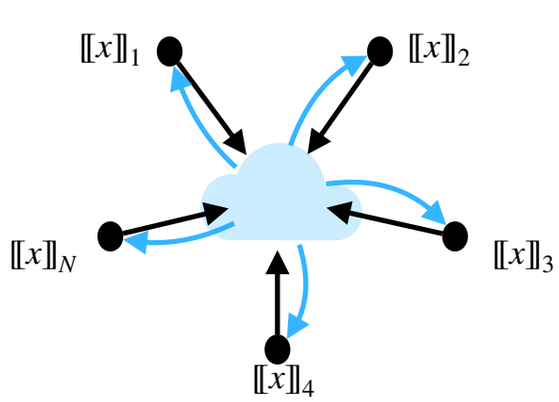
② Run MPC in their head



$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$
…
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

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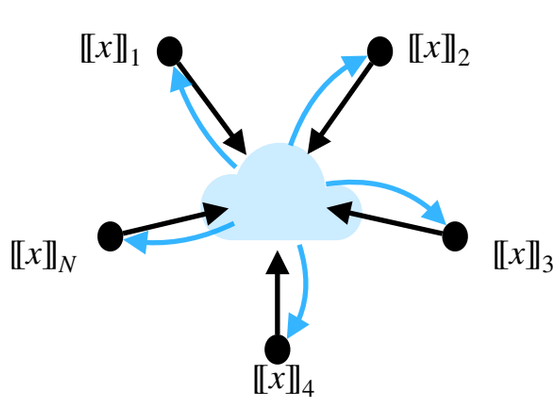

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i^*

③ Choose a random party

$i^* \leftarrow^{\$} \{1, \ldots, N\}$

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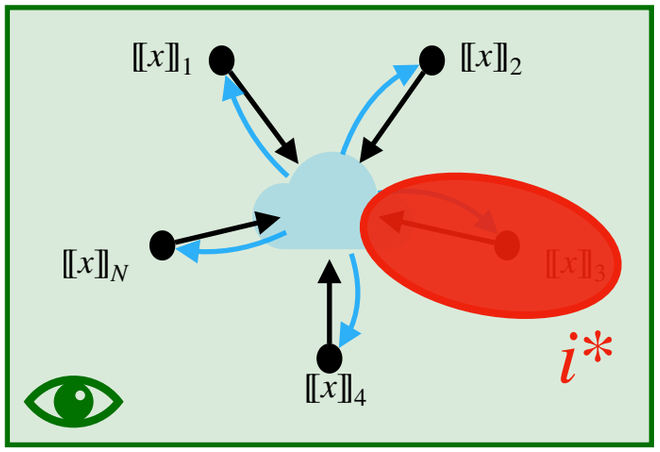

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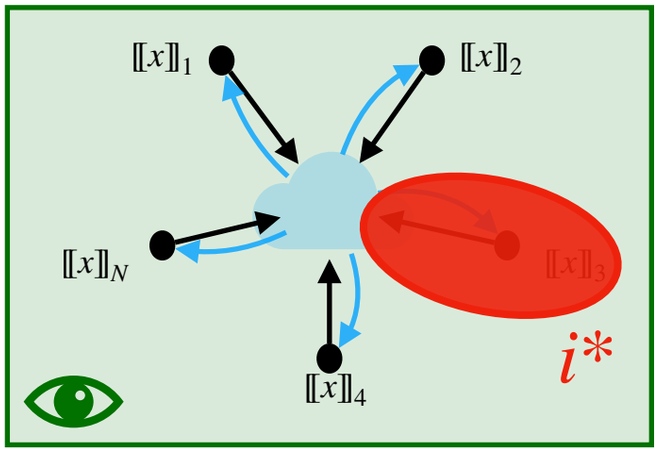

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⑤ Check $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $\tilde{g}(y, \alpha) =$ Accept

Prover

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We have $F(x) \neq y$ where

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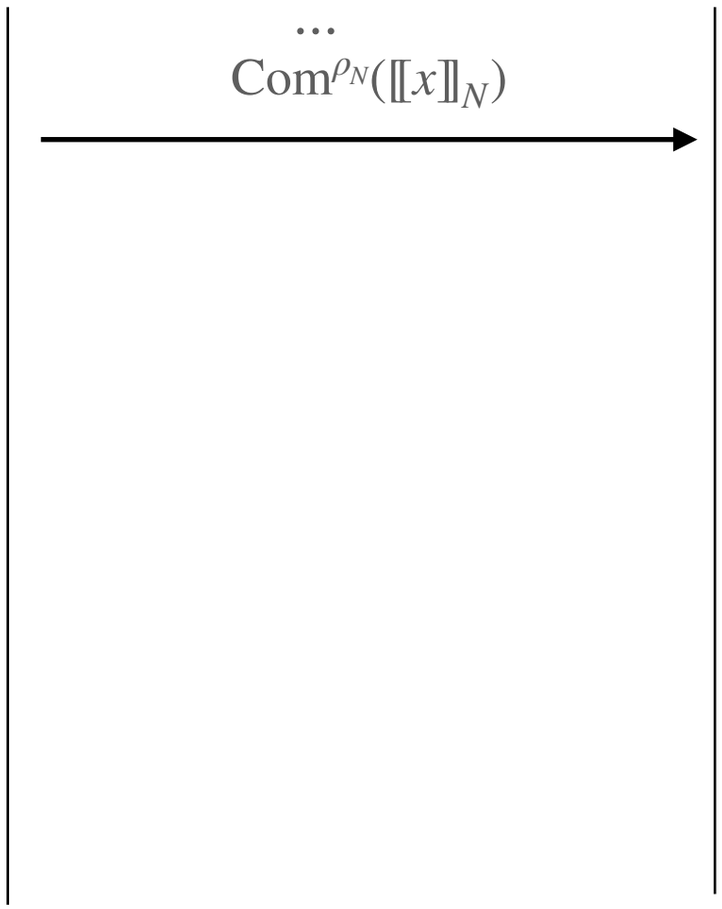

Malicious Prover Verifier

MPCitH transform

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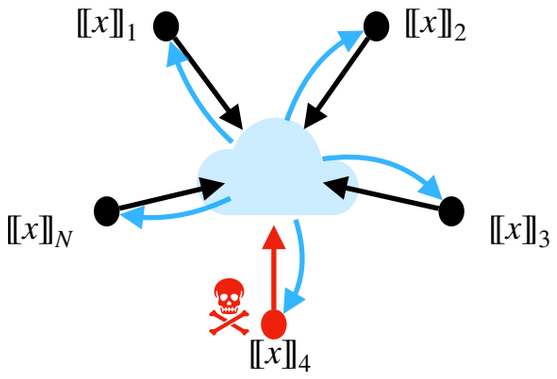

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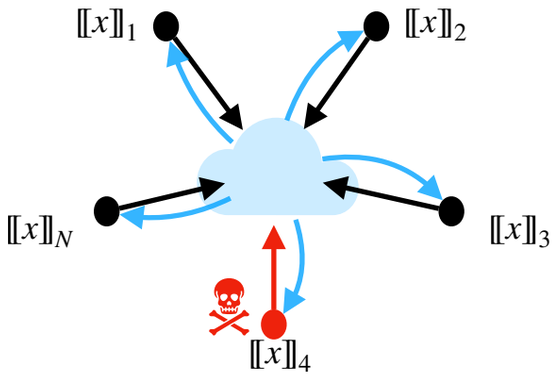

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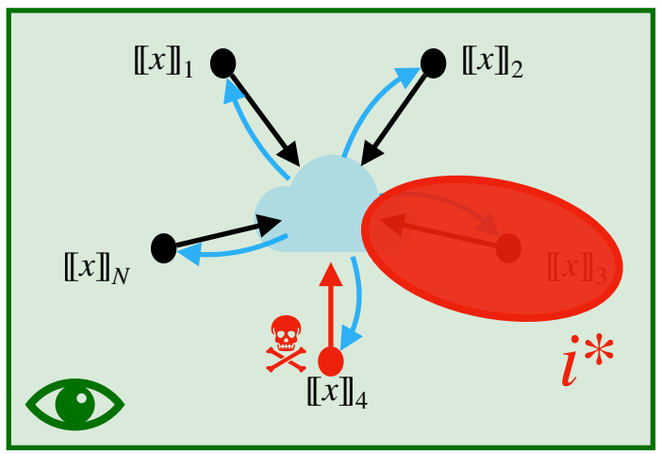

④ Open parties $\{1,\ldots,N\}\backslash\{i^*\}$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$
$\cdots$
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

→

send broadcast
$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

→

③ Choose a random party
$i^* \leftarrow^{\$} \{1,\ldots,N\}$

i^*

←

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$

→

Malicious Prover

Verifier

MPCitH transform

① Generate and commit shares

$$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$$

We have $F(x) \neq y$ where

$$x := [\![x]\!]_1 + \ldots + [\![x]\!]_N$$

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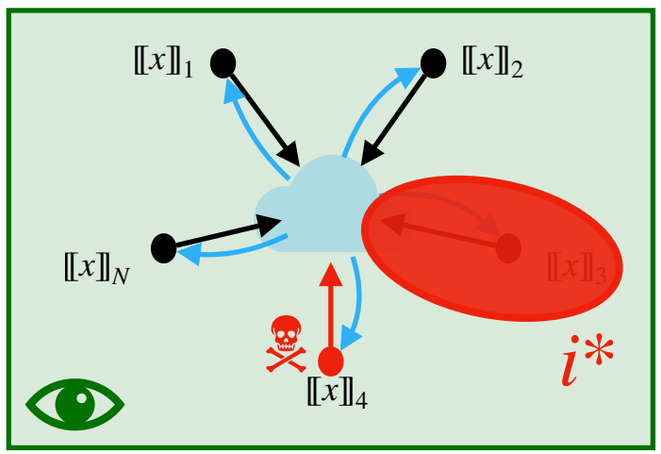

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- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $\tilde{g}(y, \alpha) =$ Accept

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Cheating detected!

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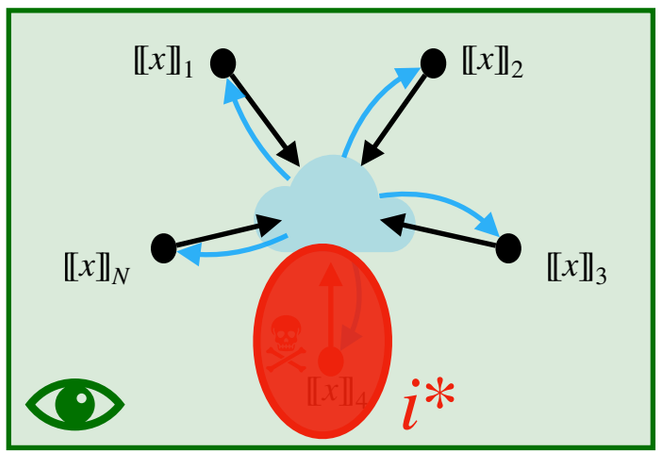

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Malicious Prover

Verifier

Seems OK.

MPCitH transform

- **Zero-knowledge** $\iff$ MPC protocol is $(N-1)$-private

MPCitH transform

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- **Soundness:**

$$\begin{aligned}&\mathbb{P}(\text{malicious prover convinces the verifier})\\ &\qquad = \mathbb{P}(\text{corrupted party remains hidden})\\ &\qquad = \frac{1}{N}\end{aligned}$$

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- **Soundness:**

$$\begin{aligned}&\mathbb{P}(\text{malicious prover convinces the verifier})\\ &\quad = \mathbb{P}(\text{corrupted party remains hidden})\\ &\quad = \frac{1}{N}\end{aligned}$$

- **Parallel repetition**

Protocol repeated τ times in parallel → soundness error $\left(\frac{1}{N}\right)^{\tau}$

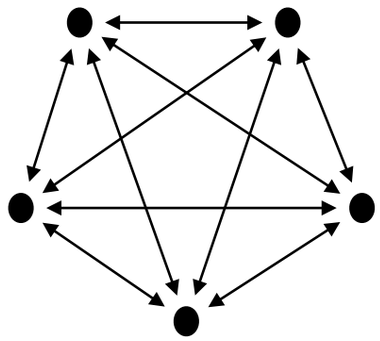
From MPC-in-the-Head to signatures

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

Multiparty computation (MPC)



Input sharing $[\![x]\!]$

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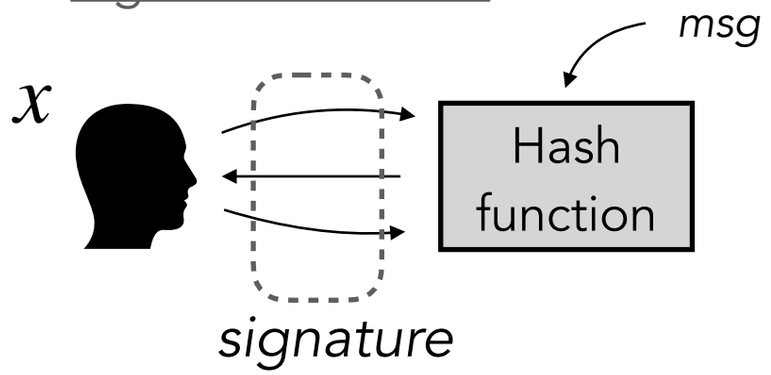
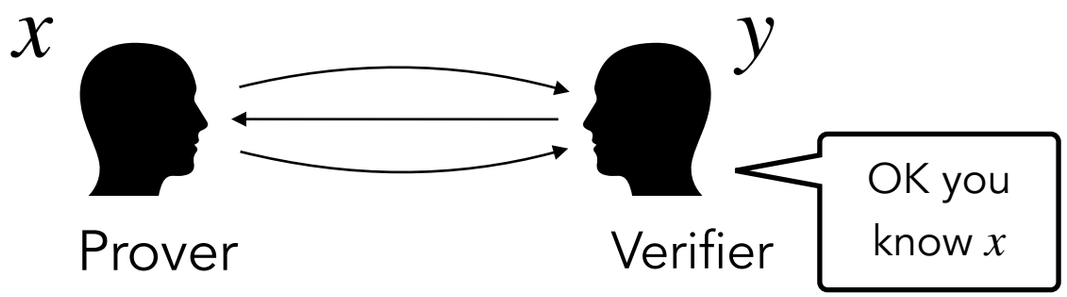
Signature scheme



Zero-knowledge proof



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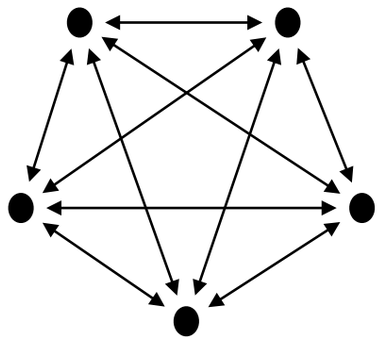

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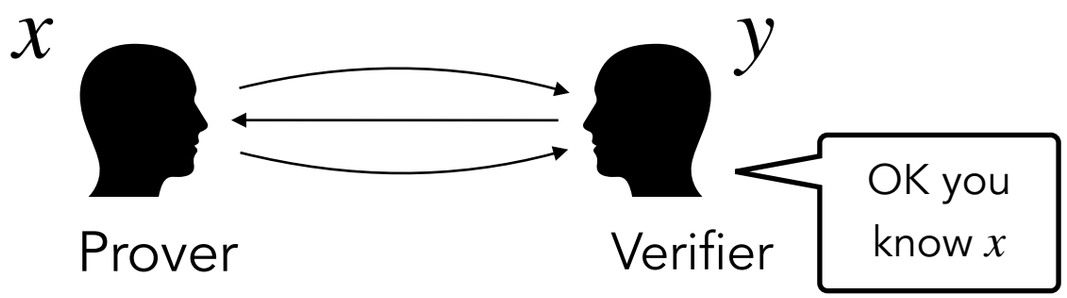
MPC-in-the Head transform

Zero-knowledge proof



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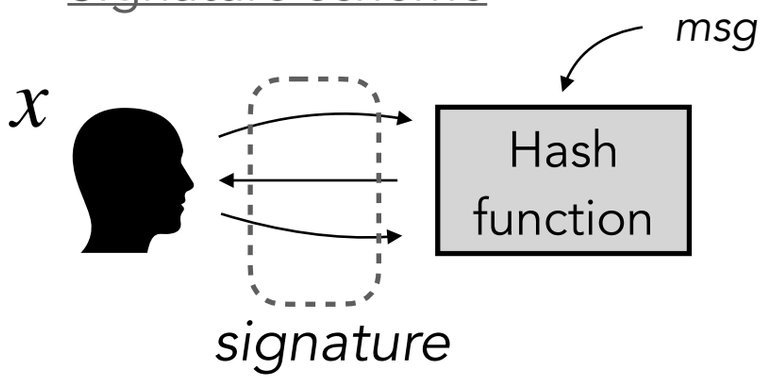

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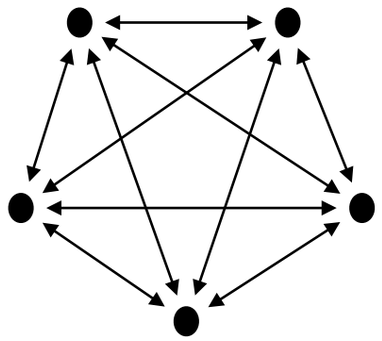

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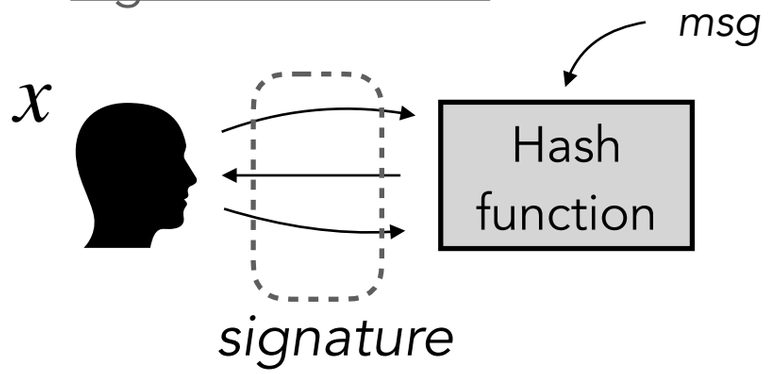

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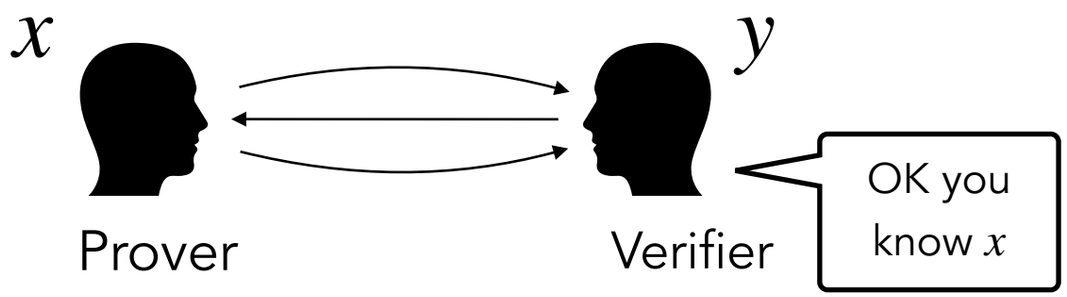

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Three approaches:

- Rely on standard symmetric primitives
  - AES: *BBQ* (2019), *Banquet* (2021), *Limbo-Sign* (2021), *Helium+AES* (2022)

One-way function

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Three approaches:

- Rely on standard symmetric primitives
- Rely on MPC-friendly symmetric primitives
  - LowMC: *Picnic1* (2017), *Picnic2* (2018), *Picnic3* (2020)
  - Rain: *Rainier* (2021), *BN++Rain* (2022)
  - AIM: *AIMer* (2022)

One-way function

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E.g. AES, MQ system,
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Three approaches:

- Rely on standard symmetric primitives
- Rely on MPC-friendly symmetric primitives
- Rely on well-known hard problems *(non-exhaustive list)*
  - Syndrome Decoding: *SDitH* (2022), *RYDE* (2023)
  - MinRank: *MiRitH* (2022), *MIRA* (2023)
  - Multivariate Quadratic: *MQOM* (2023), *Biscuit* (2023)
  - Permuted Kernel: *PERK* (2023)

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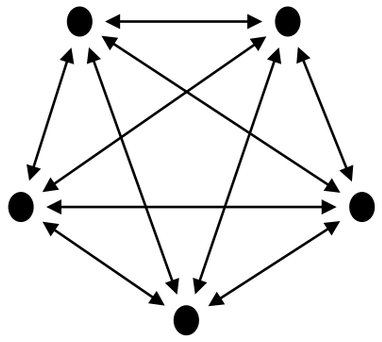

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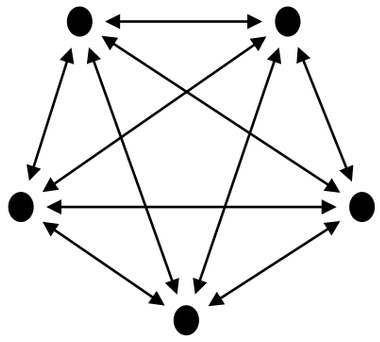

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Expressed as an arithmetic circuit, enabling us to use existing MPCitH-based proof systems (as BN++)

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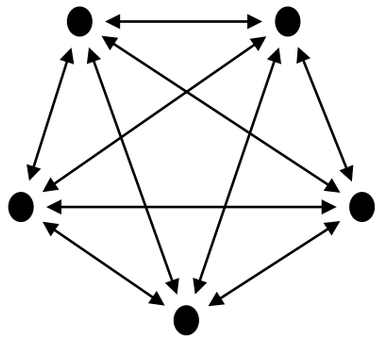

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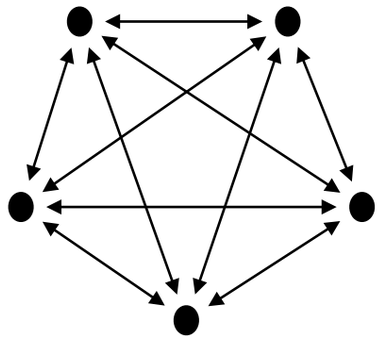

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Should be rephrased to achieve interesting performances

Example (RYDE): how to check that a vector $x \in \mathbb{F}_{q^m}^n$ has a rank weight smaller than some public bound r ?

By checking that $x_1, \ldots, x_n$ are roots of a degree-q^r q-polynomial $\sum_{i=0}^{r} a_i X^{q^i}$.

[Fen22] Feneuil. "Building MPCitH-based Signatures from MQ, MinRank, Rank SD and PKP" (ePrint 2022/1512)

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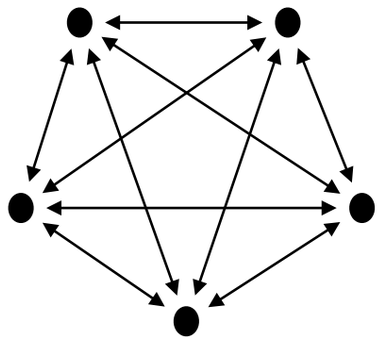

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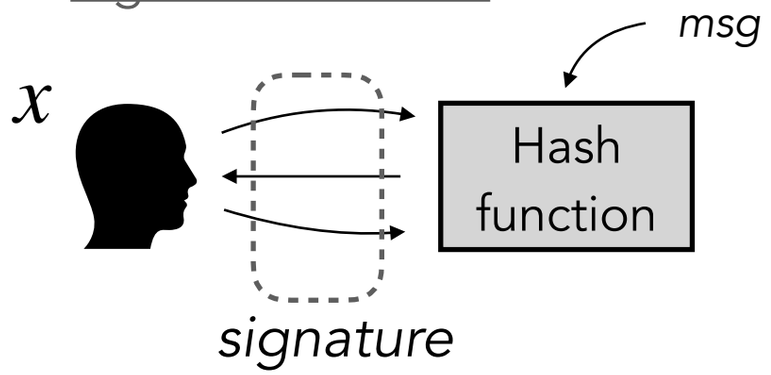
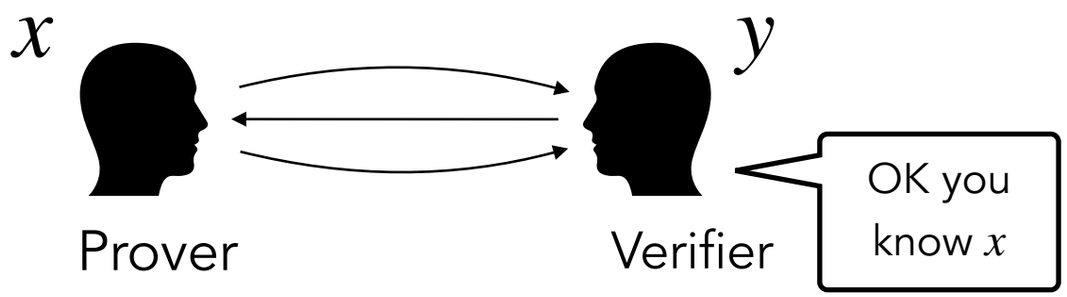


Zero-knowledge proof



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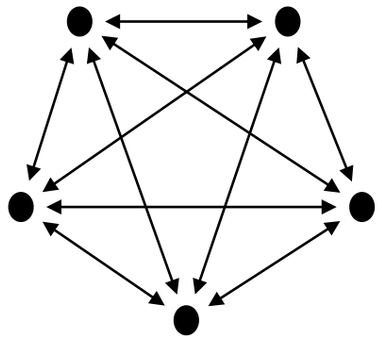

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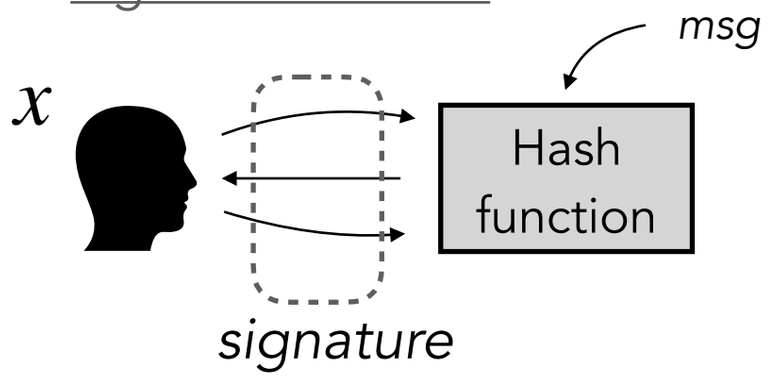

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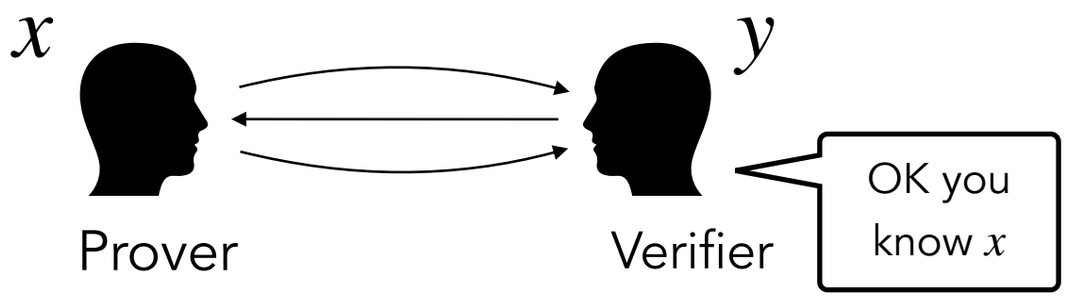

Fiat-Shamir transform

Should take [KZ20] attack into account (when there are more than 3 rounds)!

[KZ20] Kales, Zaverucha. "An attack on some signature schemes constructed from five-pass identification schemes" (CANS20)

Optimisations and variants

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Field $GF(251)$

With `SDitH-L1-gf251` as example.

NIST Category I

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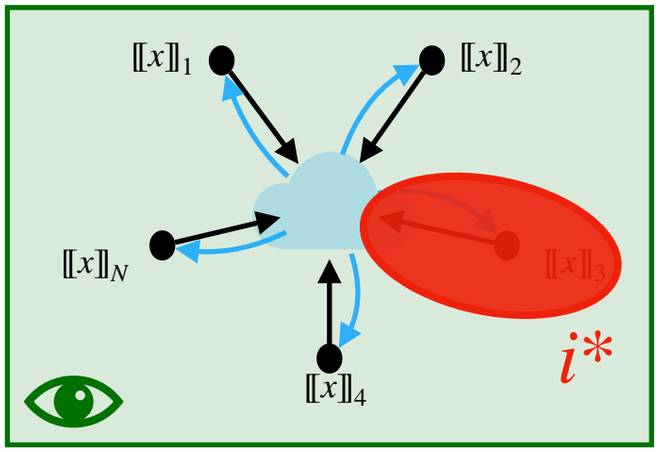

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Verifier

Naive MPCitH transformation

Size of the broadcast (per party)

Size of a commitment digest

Size of the MPC input (per party)

$$\text{Size} \approx \tau \cdot \left(N \cdot 2\lambda + N \cdot |\alpha| + (N-1) \cdot |x| \right)$$

Number of repetitions to achieve the desired security level

$$\tau \approx \frac{\lambda}{\log_2 N}$$

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`SDitH-L1-gf251`:

the input x of the MPC protocol is around **323** bytes,
The broadcast value α of the MPC protocol is around **36** bytes.

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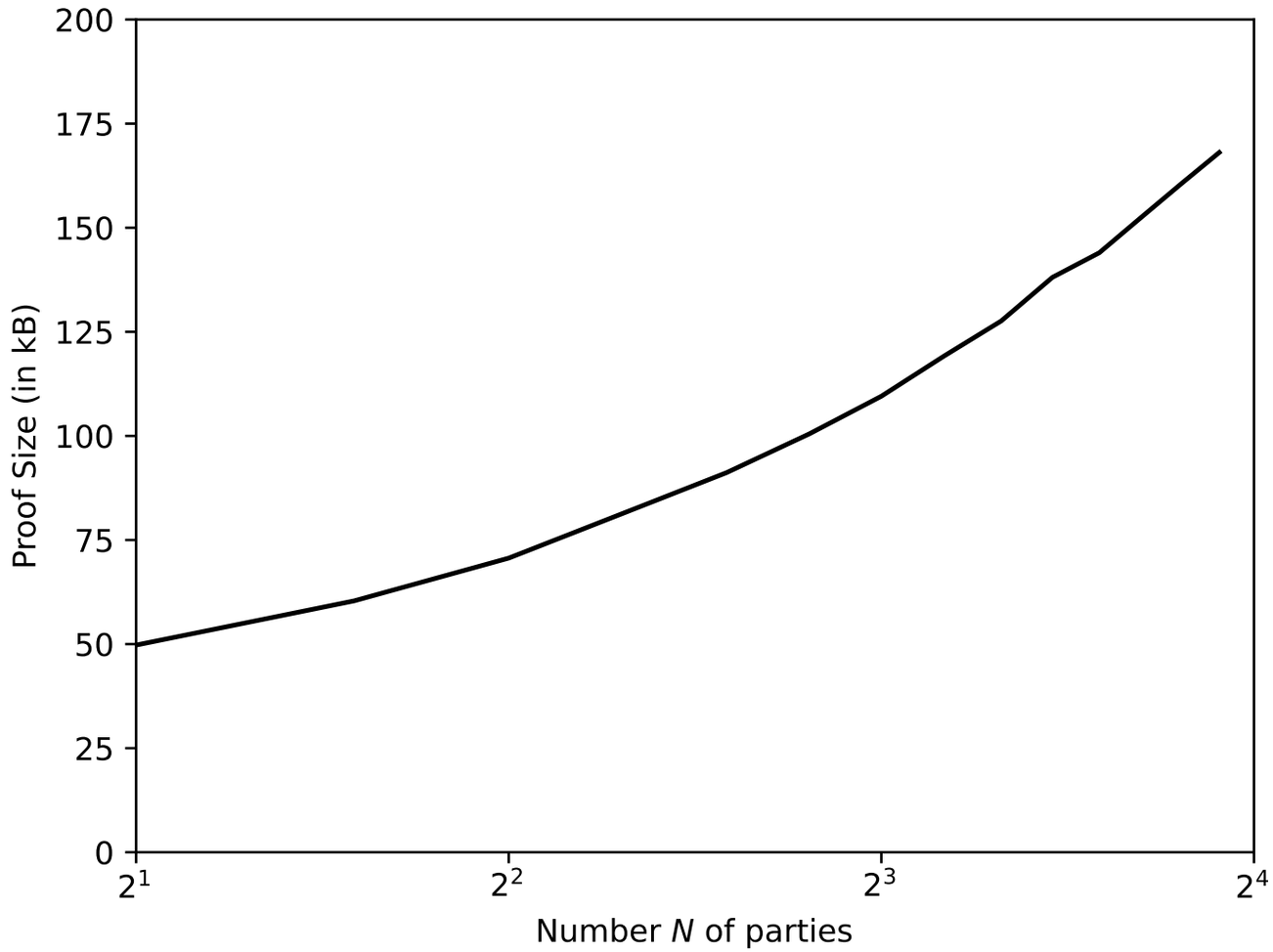

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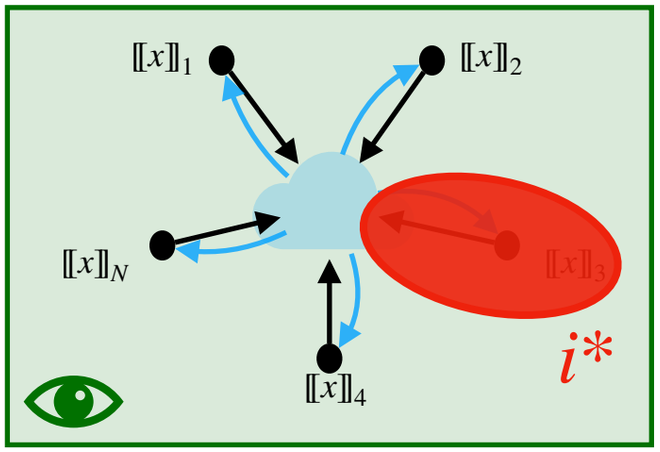

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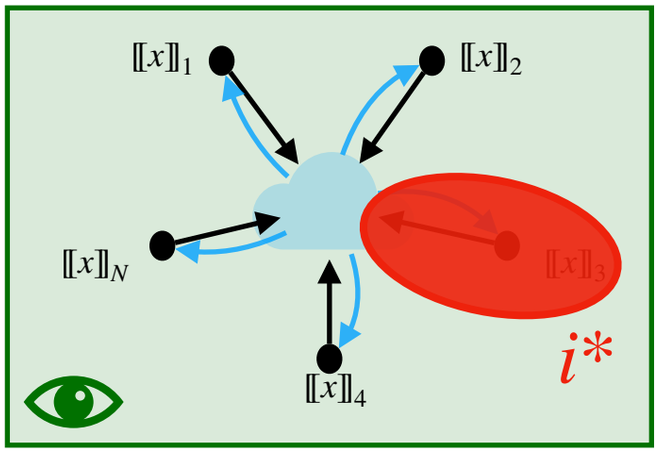

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Prover

$h_1 = \mathrm{Hash}(\mathsf{com}_1, \ldots, \mathsf{com}_N)$ →

$h_2 = \mathrm{Hash}([\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N)$ →

③ Choose a random party

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← i^*

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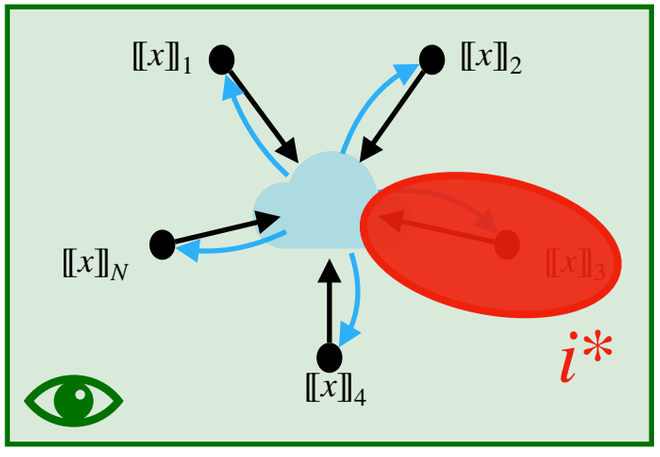

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⑤ Compute $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $\tilde{g}(y, \alpha) = \mathrm{Accept}$

Check $h_1 = \mathrm{Hash}(\mathsf{com}_1, \ldots, \mathsf{com}_N)$

Check $h_2 = \mathrm{Hash}([\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N)$

Prover

Verifier

Using a Seed Tree

[KKW18] Katz, Kolesnikov, Wang: "Improved Non-Interactive Zero Knowledge with Applications to Post-Quantum Signatures" (CCS 2018)

$$x = [\![x]\!]_1 + [\![x]\!]_2 + [\![x]\!]_3 + \ldots + [\![x]\!]_{N-1} + [\![x]\!]_N$$

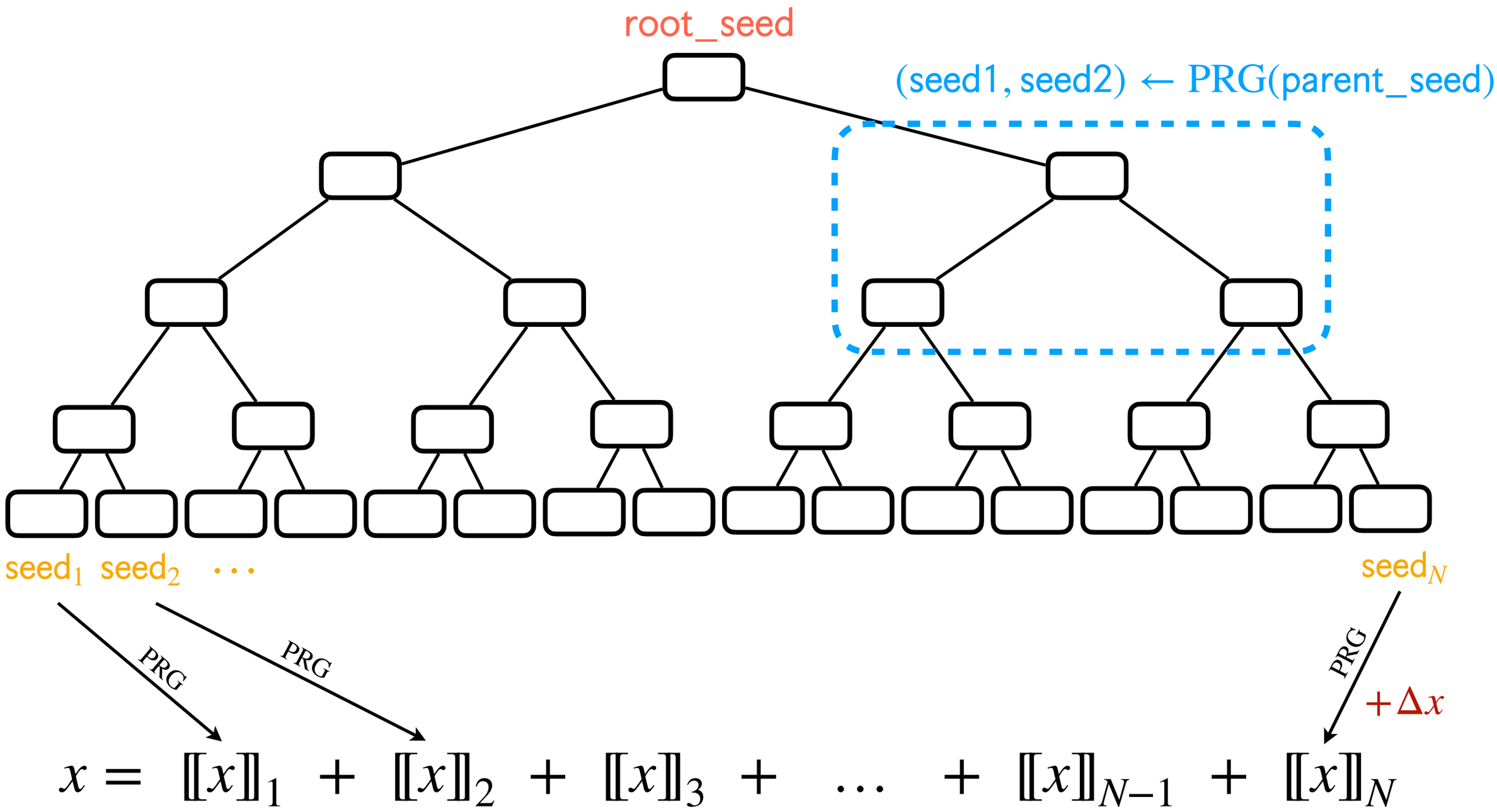
Using a Seed Tree

[KKW18] Katz, Kolesnikov, Wang: "Improved Non-Interactive Zero Knowledge with Applications to Post-Quantum Signatures" (CCS 2018)

$$x = \underset{\text{PRG}}{\overset{\text{seed}_1}{[\![x]\!]_1}} + \underset{\text{PRG}}{\overset{\text{seed}_2}{[\![x]\!]_2}} + \underset{\text{PRG}}{\overset{\text{seed}_3}{[\![x]\!]_3}} + \ldots + \underset{\text{PRG}}{\overset{\text{seed}_{N-1}}{[\![x]\!]_{N-1}}} + \underset{\text{PRG}}{\overset{\text{seed}_N}{[\![x]\!]_N}} + \Delta x$$

Using a Seed Tree

[KKW18] Katz, Kolesnikov, Wang: "Improved Non-Interactive Zero Knowledge with Applications to Post-Quantum Signatures" (CCS 2018)



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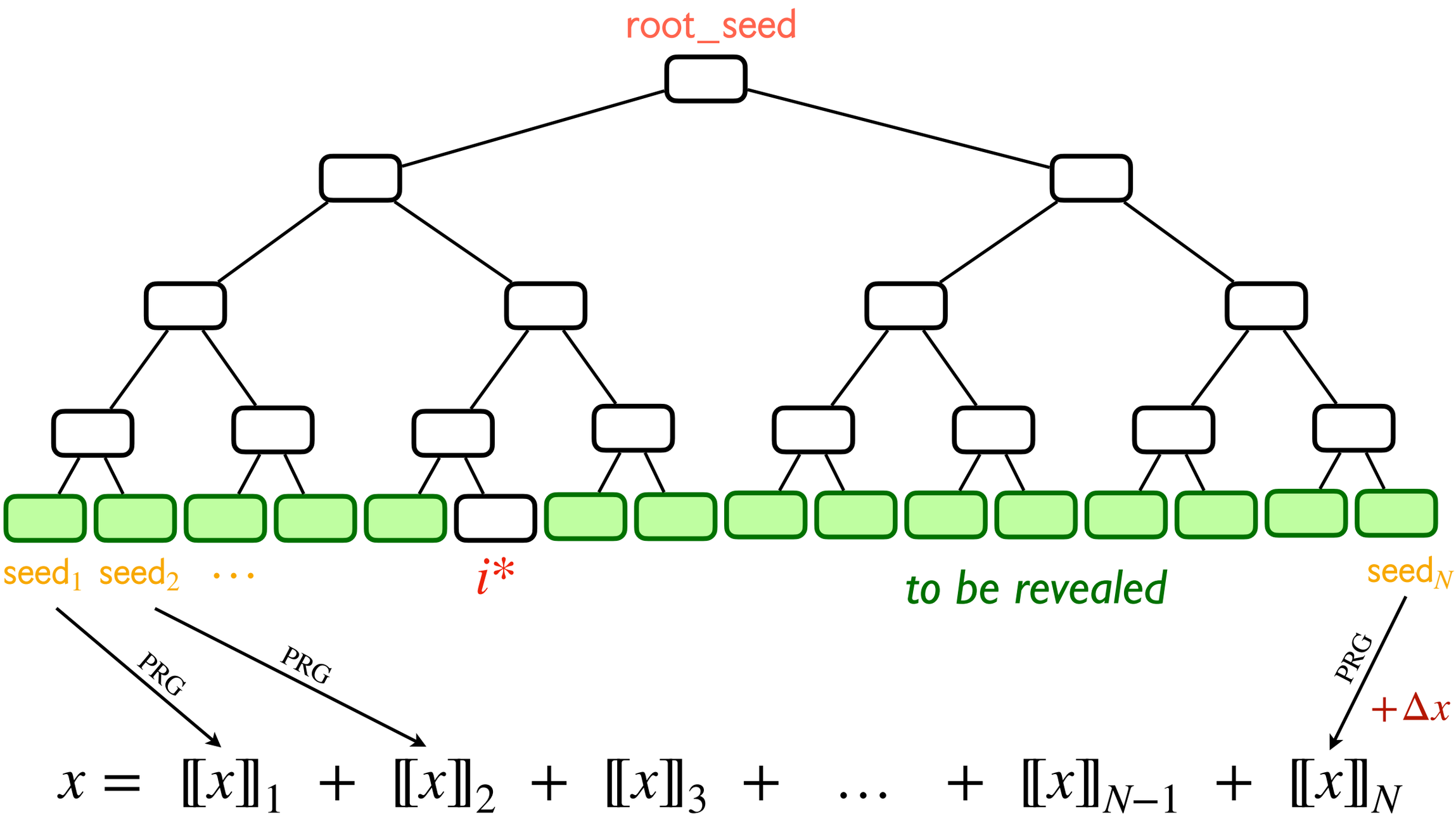

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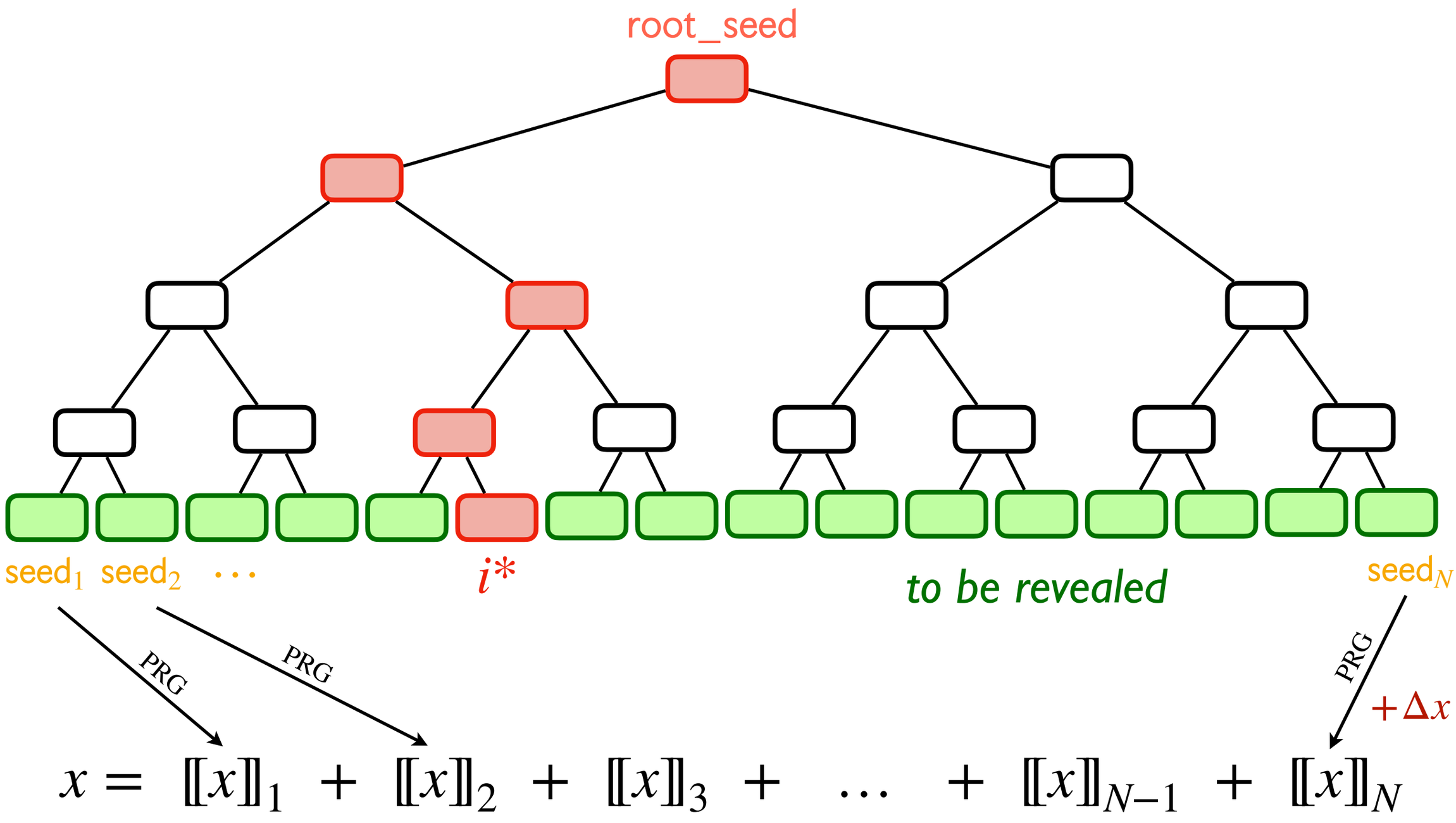

Using a Seed Tree

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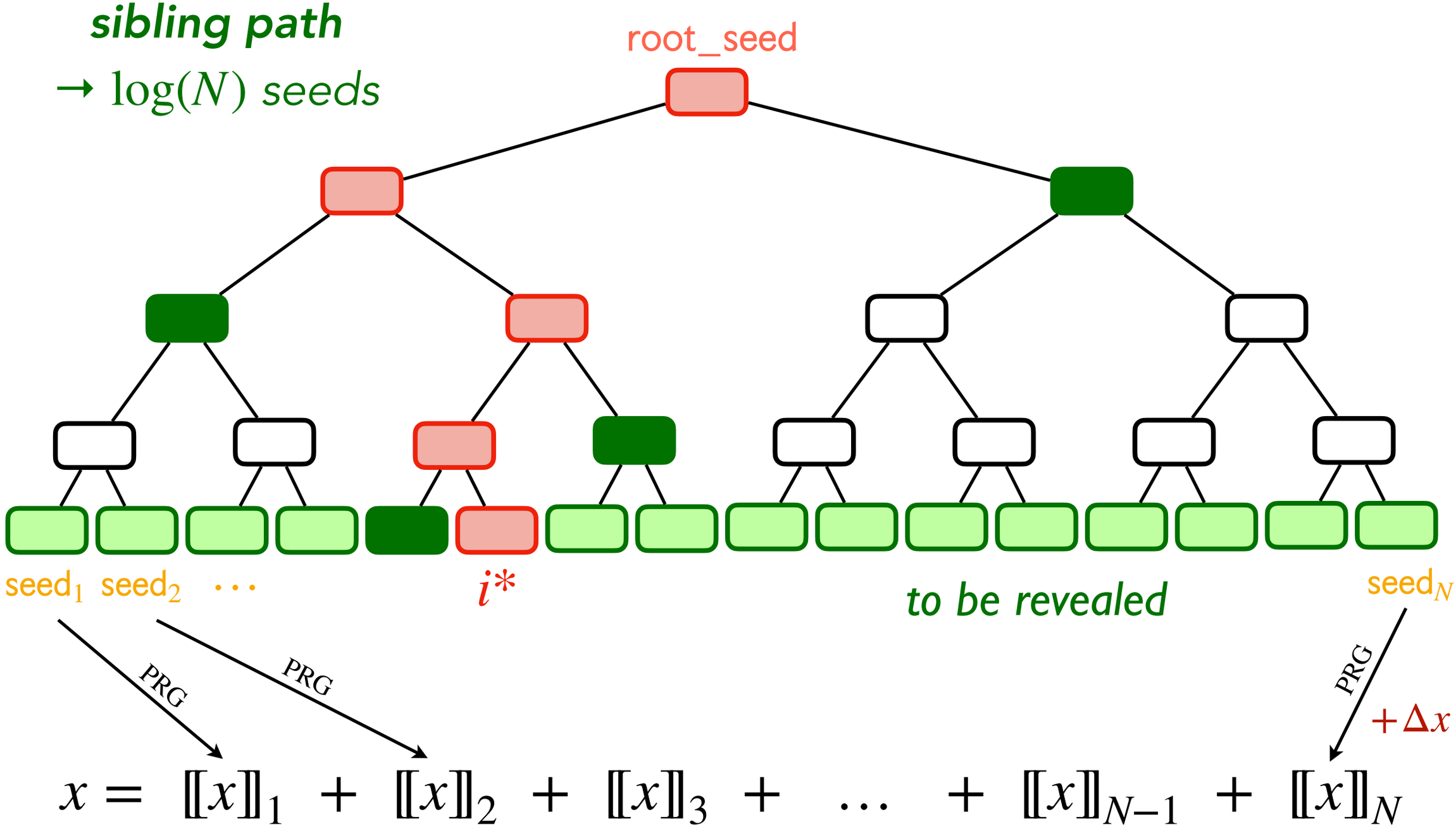
sibling path

→ $\log(N)$ *seeds*



$$x = [\![x]\!]_1 + [\![x]\!]_2 + [\![x]\!]_3 + \cdots + [\![x]\!]_{N-1} + [\![x]\!]_N$$

Traditional MPCitH transformation

Size of the broadcast (of the hidden party)

Size of the auxiliary value

Path in the seed (GGM) tree

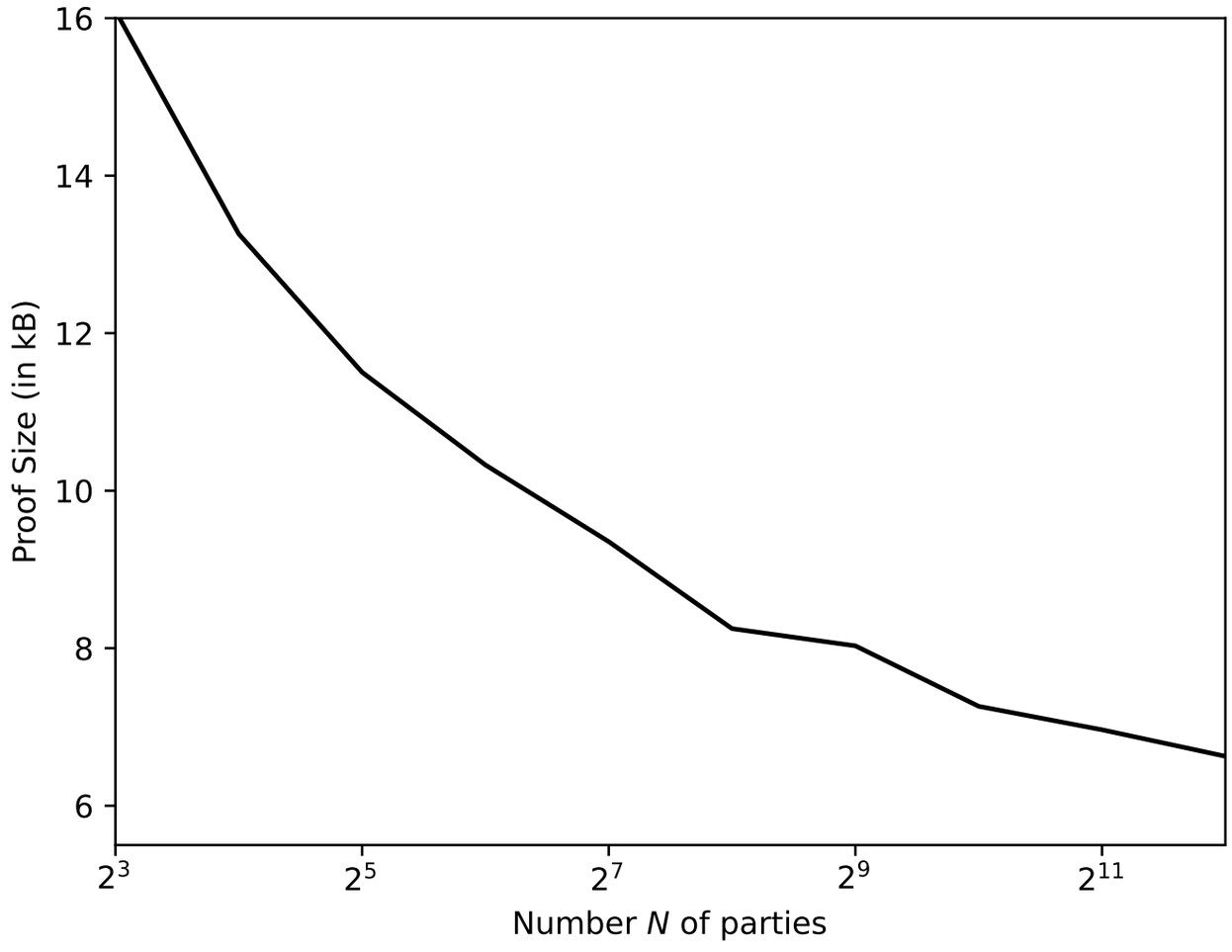
$$\text{Size} \approx \tau \cdot \left(|\Delta x| + |\alpha| + \lambda \cdot \log_2 N + 2\lambda \right)$$

Commitment of the hidden party

Number of repetitions to achieve the desired security level

$$\tau \approx \frac{\lambda}{\log_2 N}$$

Traditional MPCitH transformation



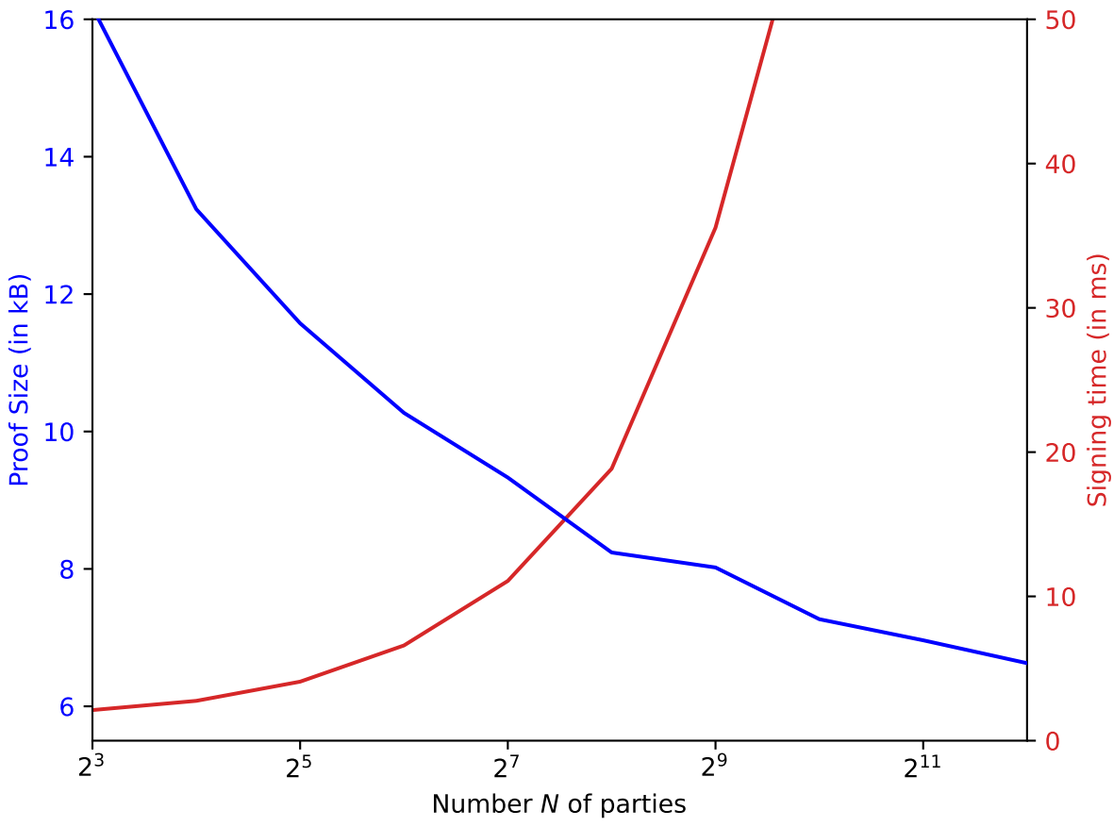
`SDitH-L1-gf251`:

the input x of the MPC protocol is around **323** bytes,

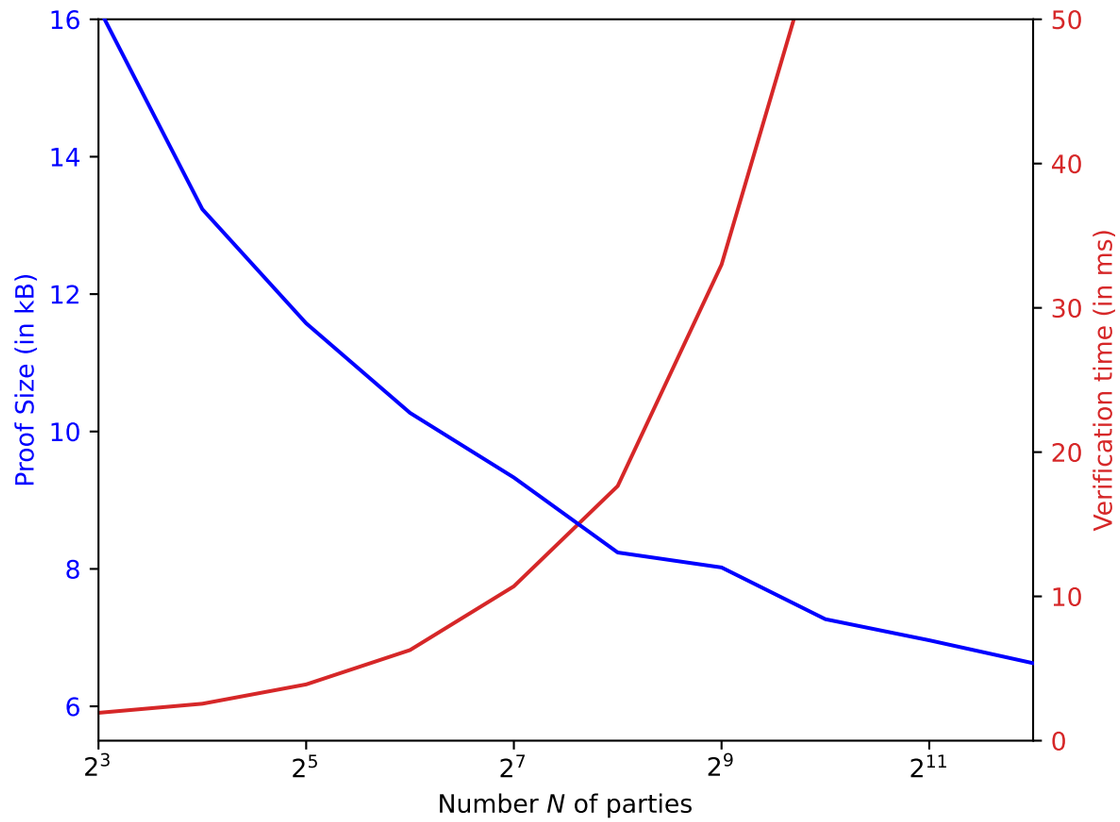
The broadcast value α of the MPC protocol is around **36** bytes.

Traditional MPCitH transformation

Signing algorithm



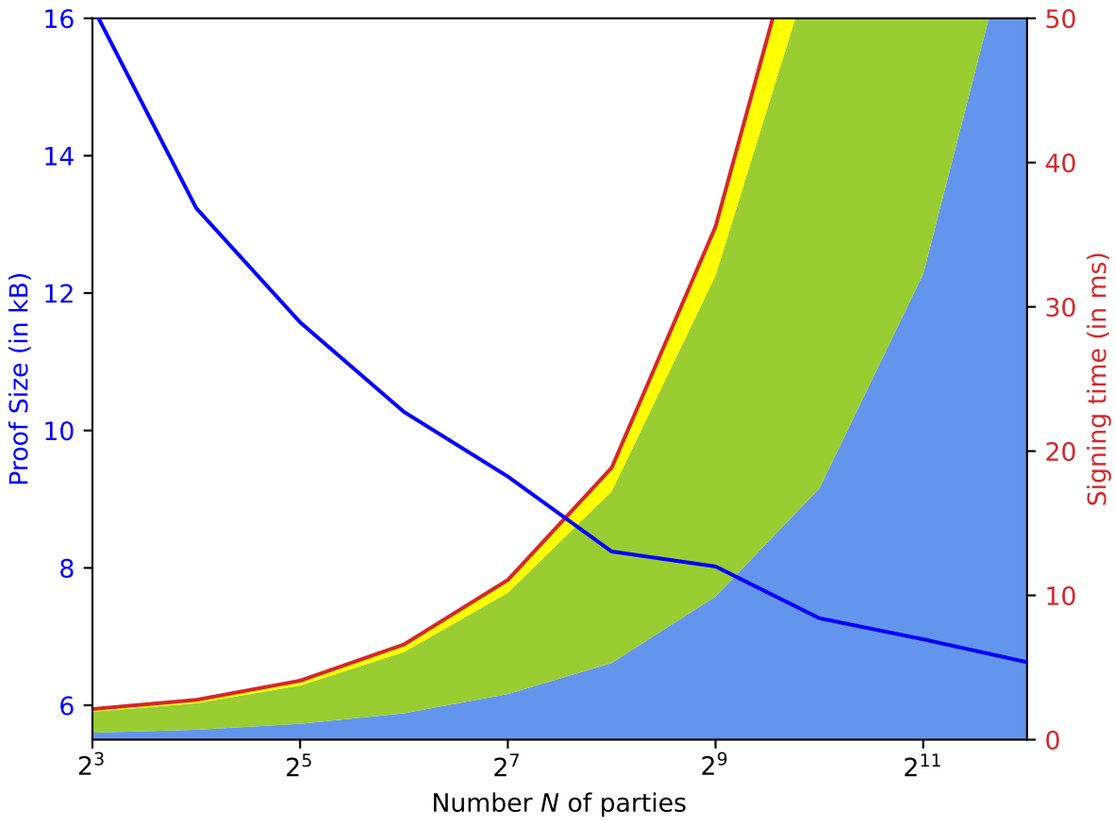
Verification algorithm



Running times @3.80Ghz

Traditional MPCitH transformation

Signing algorithm



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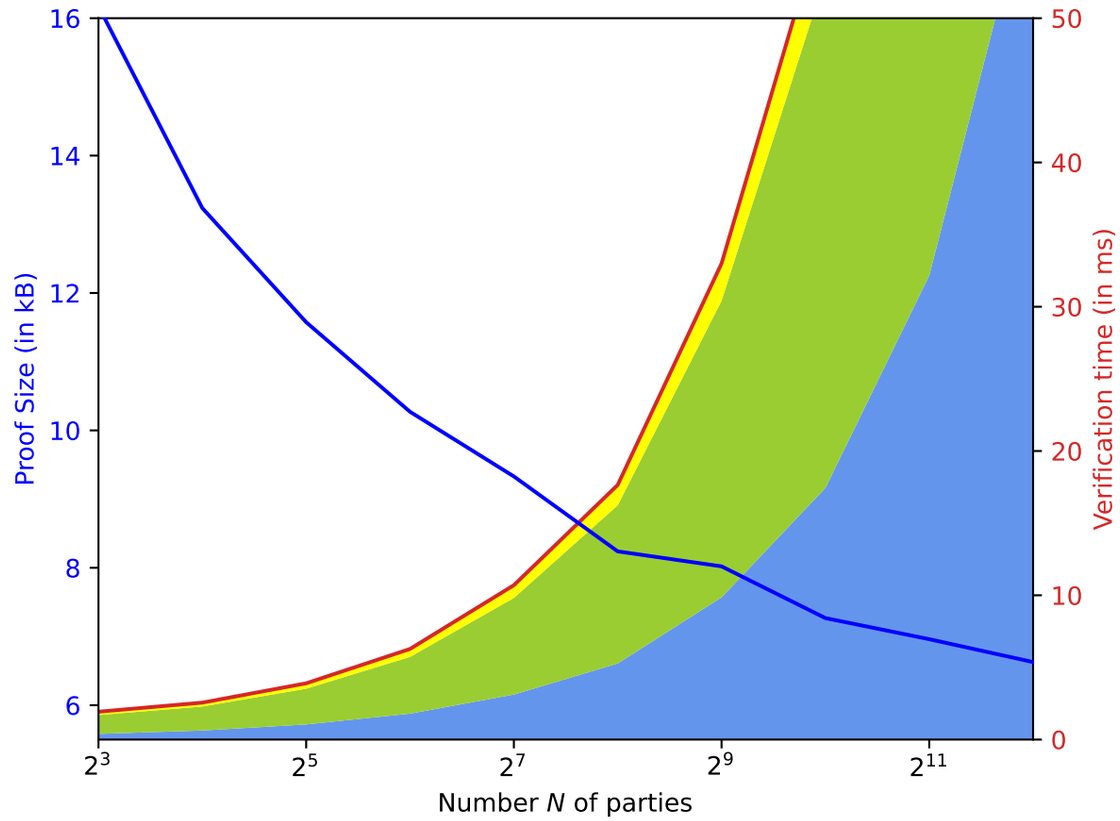
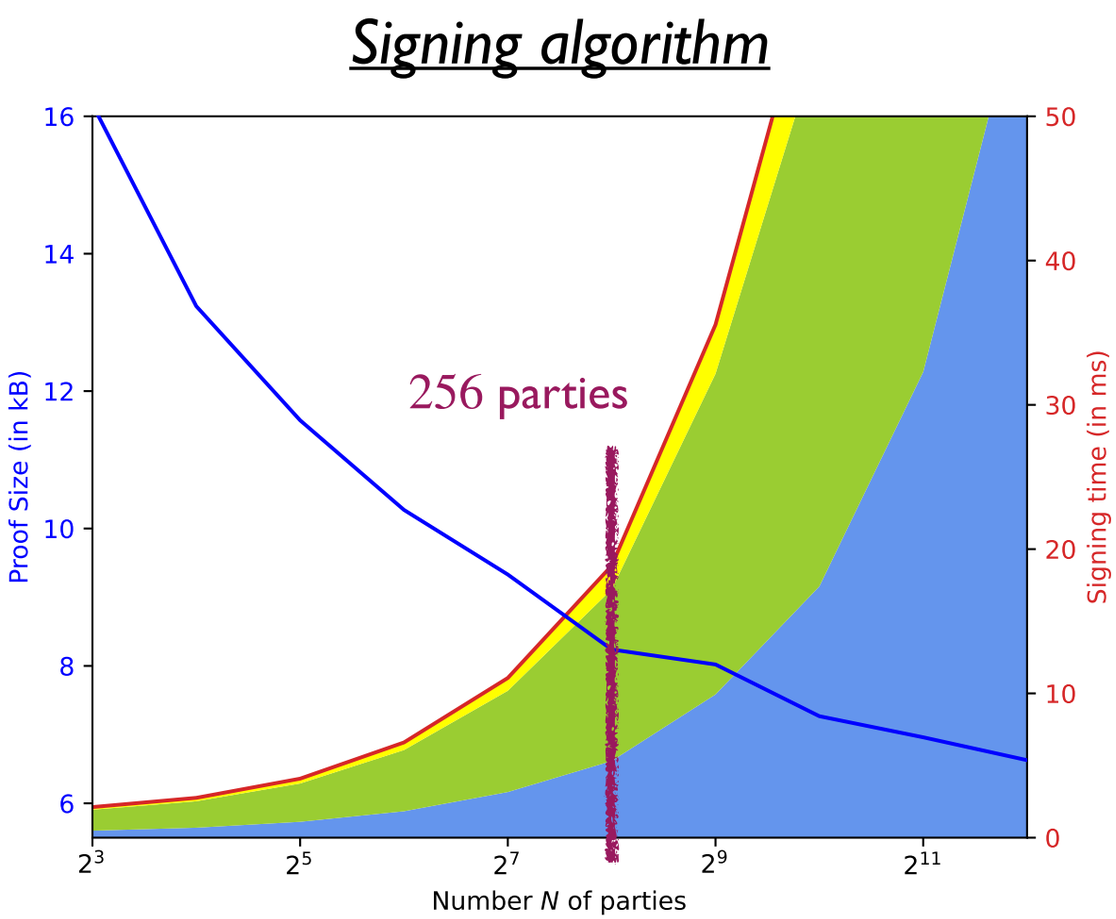
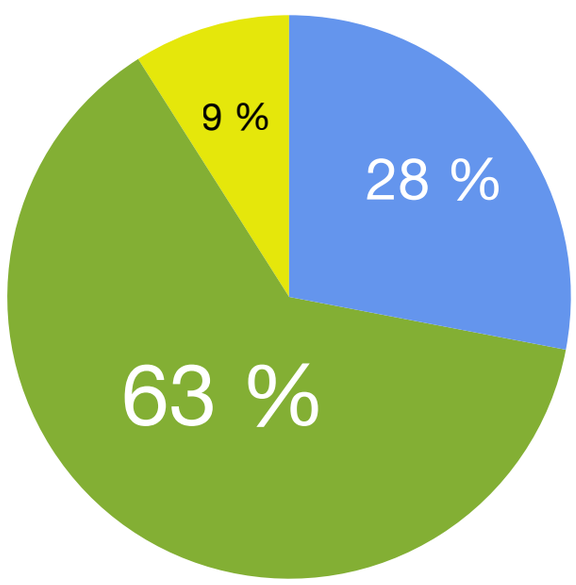


Symmetric
MPC Emulation
Misc

Running times @3.80Ghz

Traditional MPCitH transformation





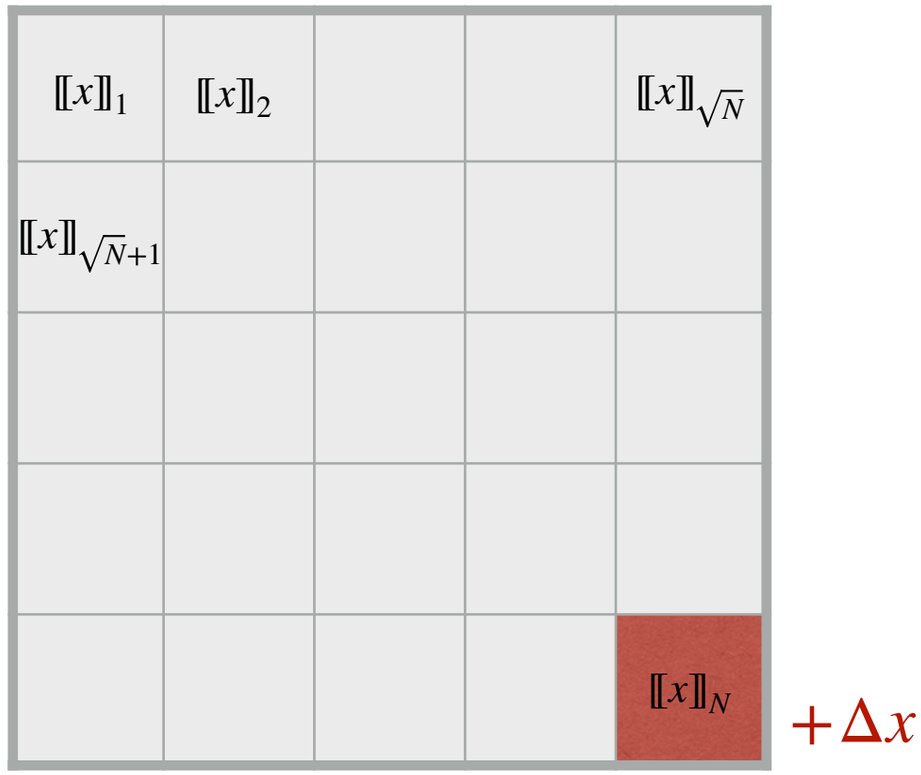
Symmetric
MPC Emulation
Misc

Signing time
for $N := 256$ parties
(19 ms)

Running times @3.80Ghz

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)



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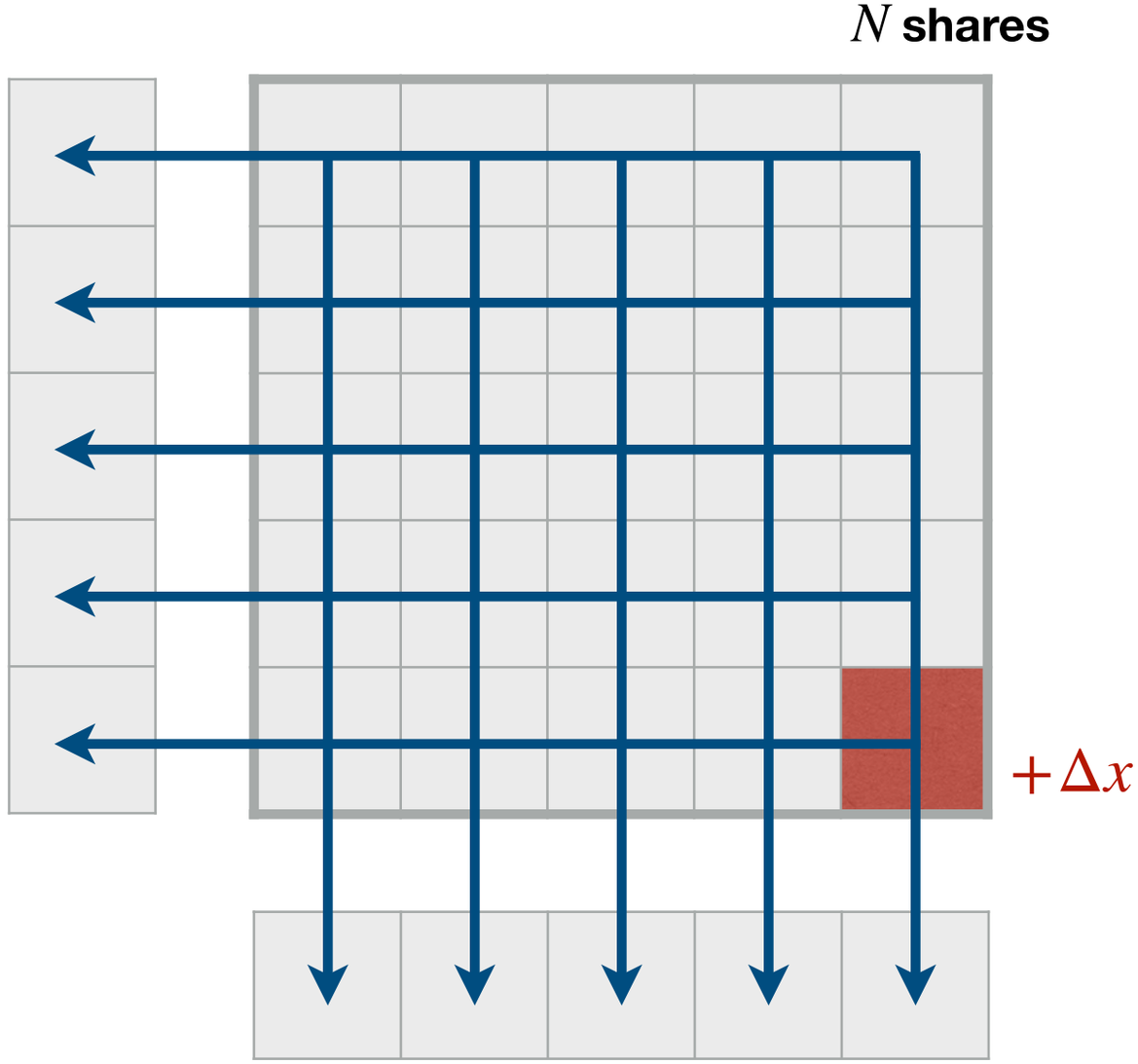

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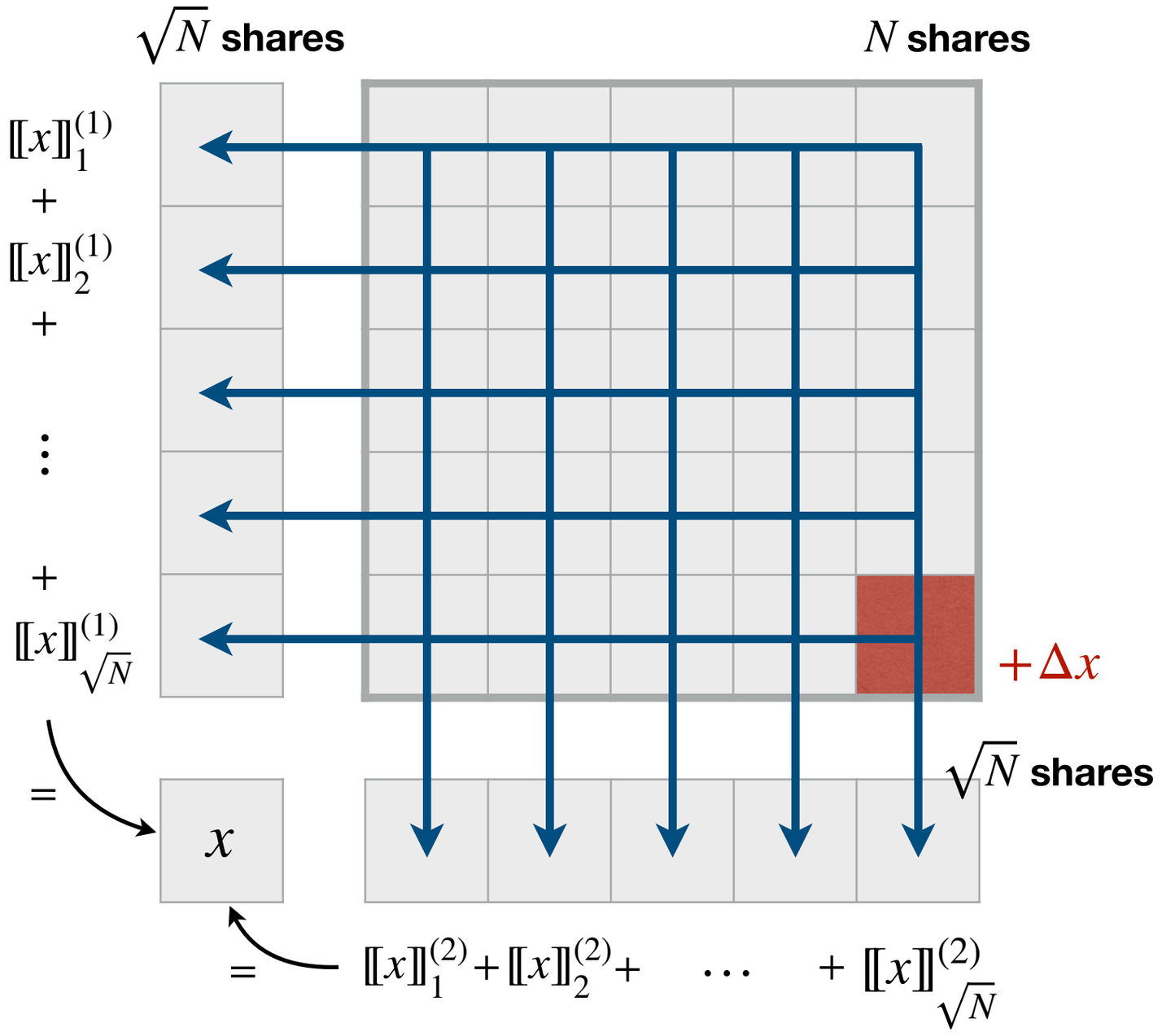

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- Emulating the N-party protocol with inputs $[\![x]\!]_1, \ldots, [\![x]\!]_N$
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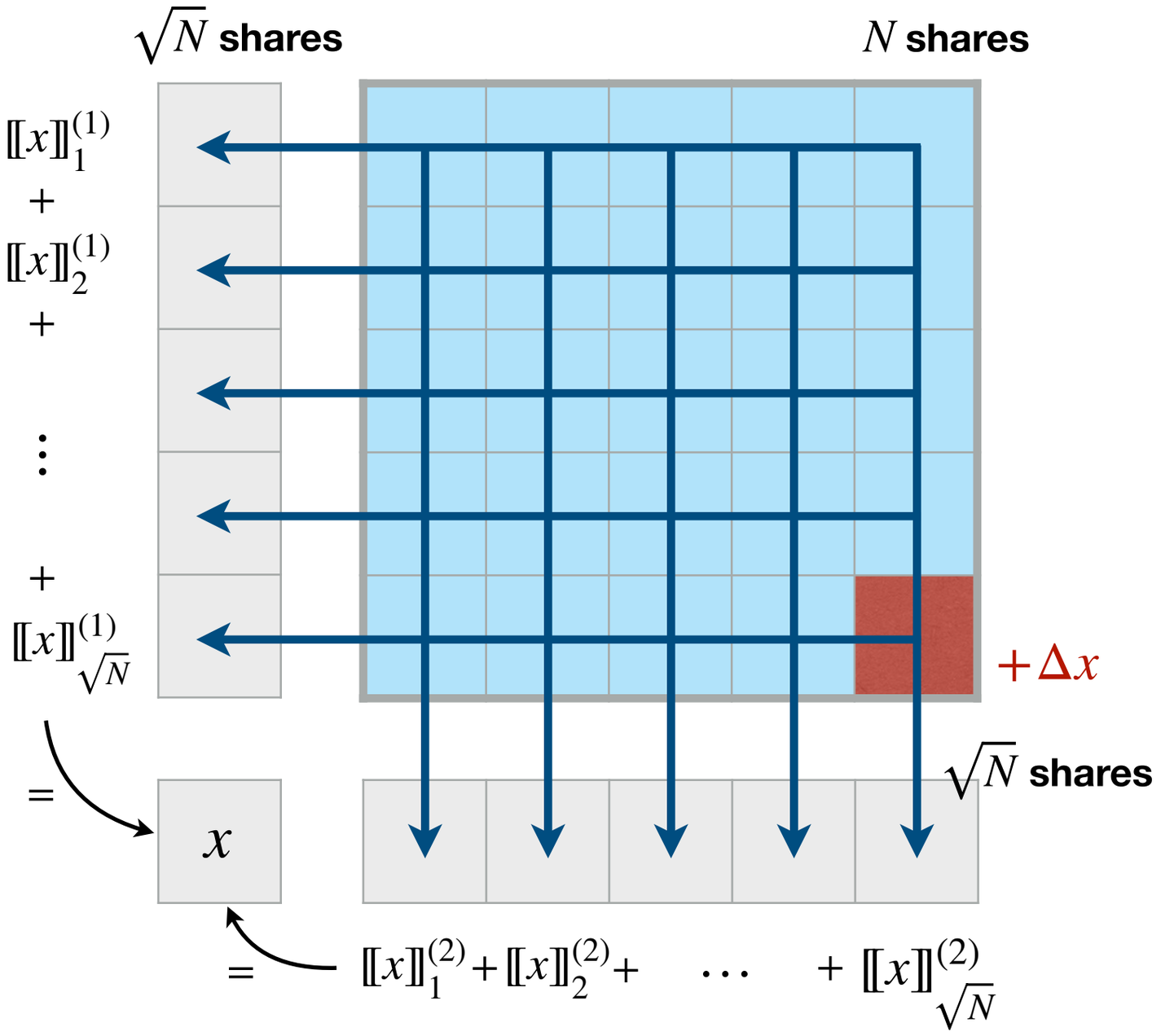

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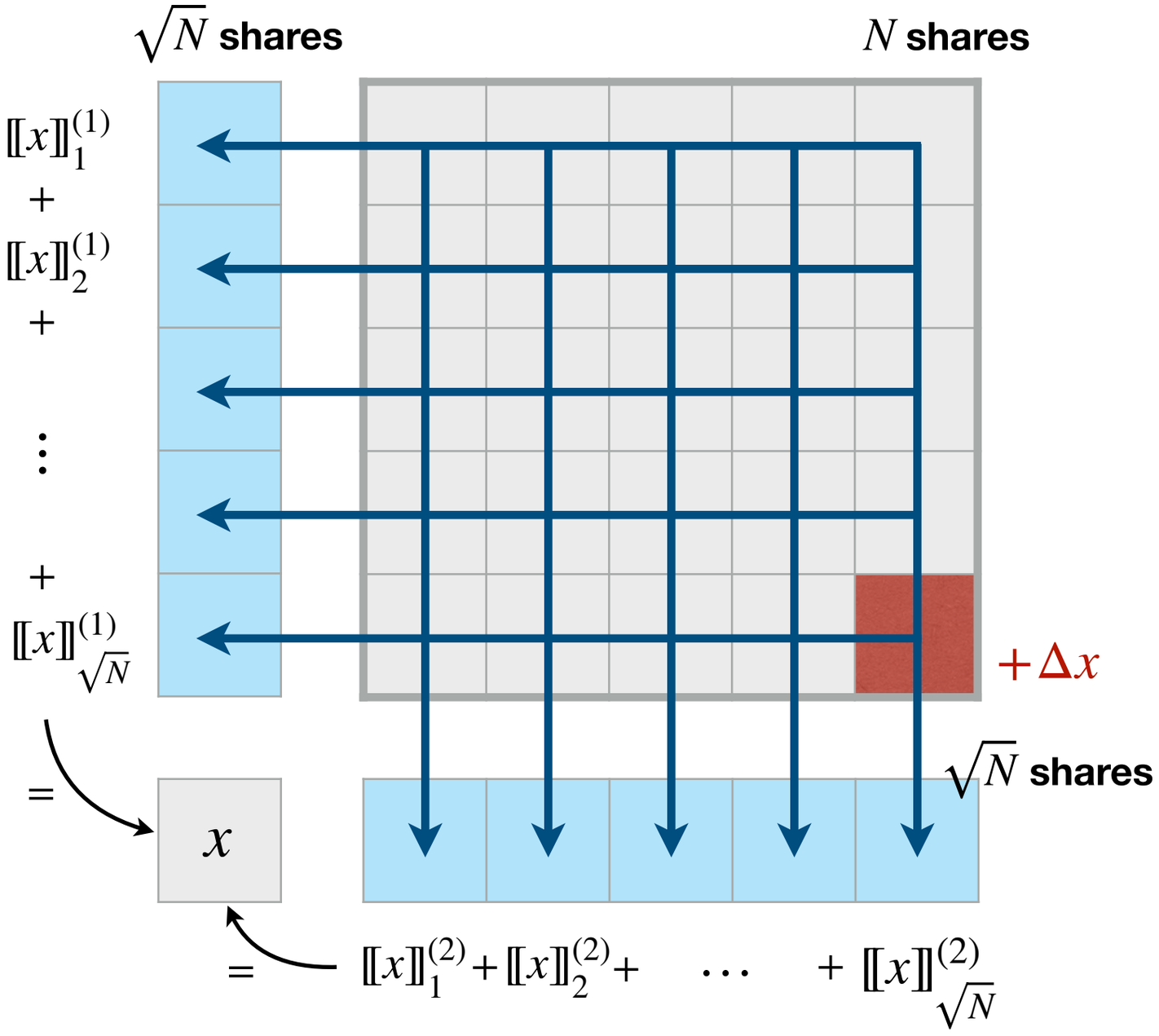
Hypercube technique:

- Emulating the $\sqrt{N}$-party protocol with inputs $[\![x]\!]_1^{(1)}, \ldots, [\![x]\!]_{\sqrt{N}}^{(1)}$
- Emulating the $\sqrt{N}$-party protocol with inputs $[\![x]\!]_1^{(2)}, \ldots, [\![x]\!]_{\sqrt{N}}^{(2)}$
- Chance of cheating

$$\left(\frac{1}{\sqrt{N}}\right)^2 \to \frac{1}{N}$$



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N evals

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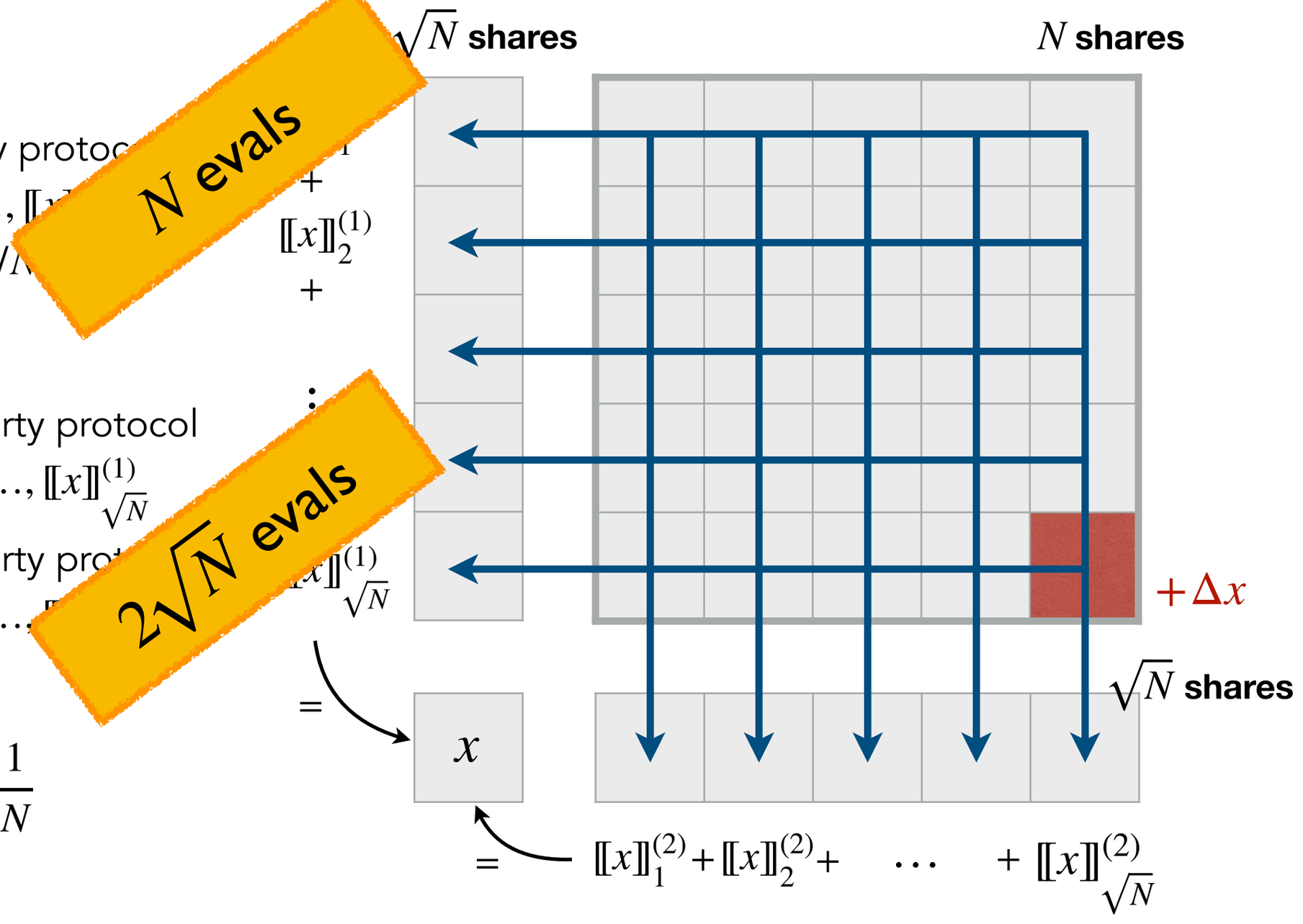
$2\sqrt{N}$ evals



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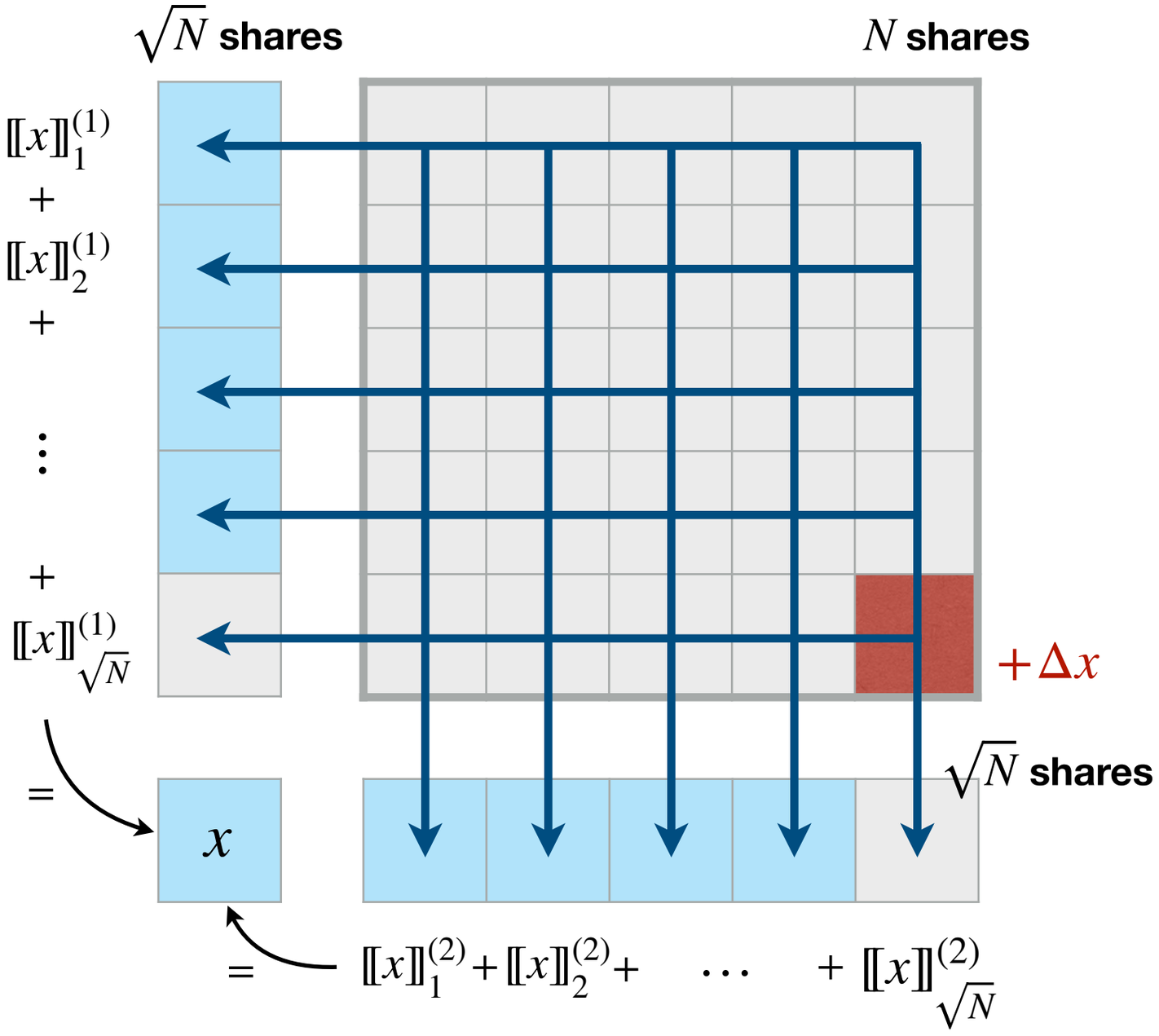

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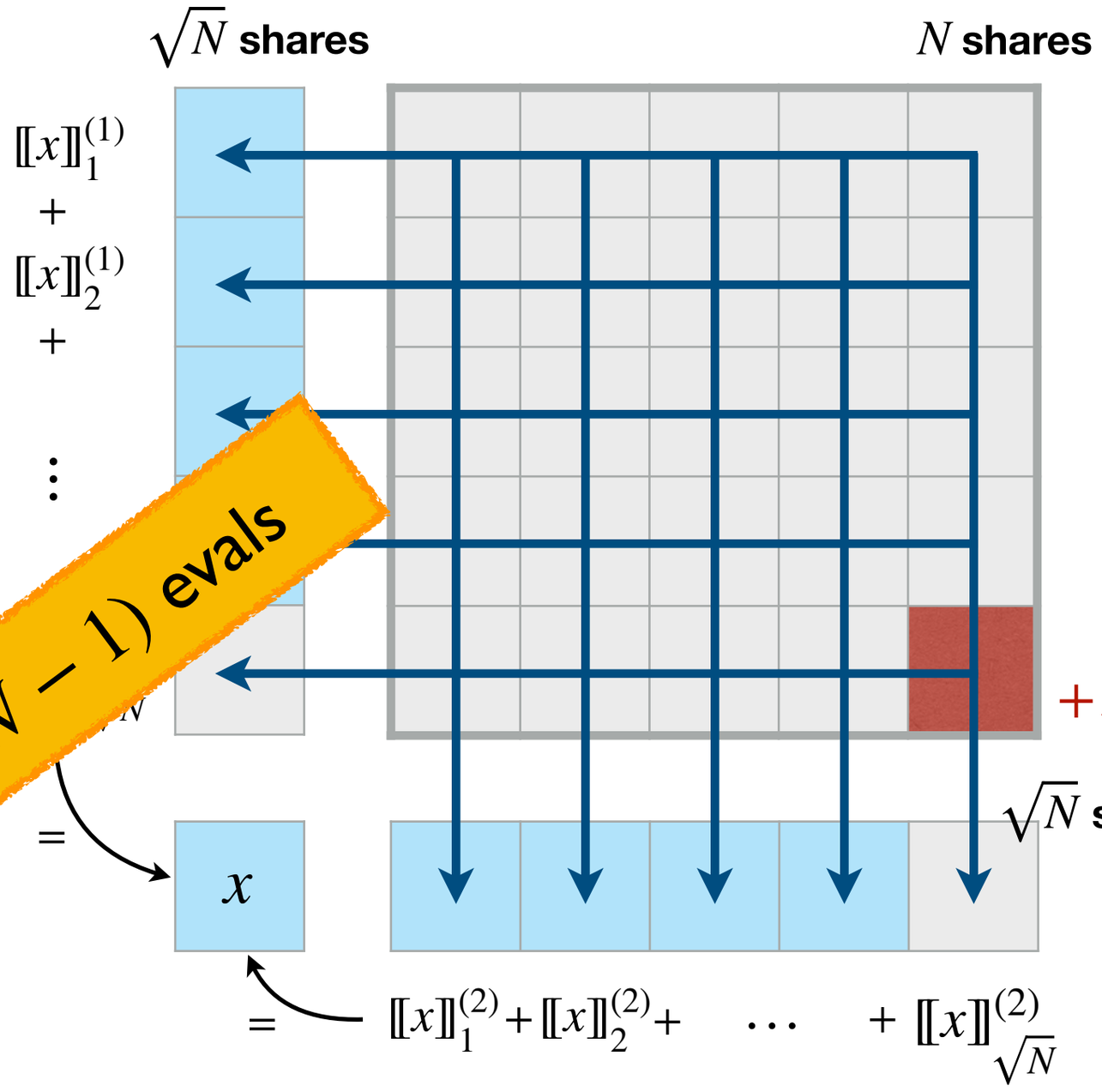
- Chance of cheating $\left(\frac{1}{\sqrt{N}}\right)^2 \rightarrow \frac{1}{N}$



The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

Previous slide: square of side $\sqrt{N}$

The Hypercube Technique

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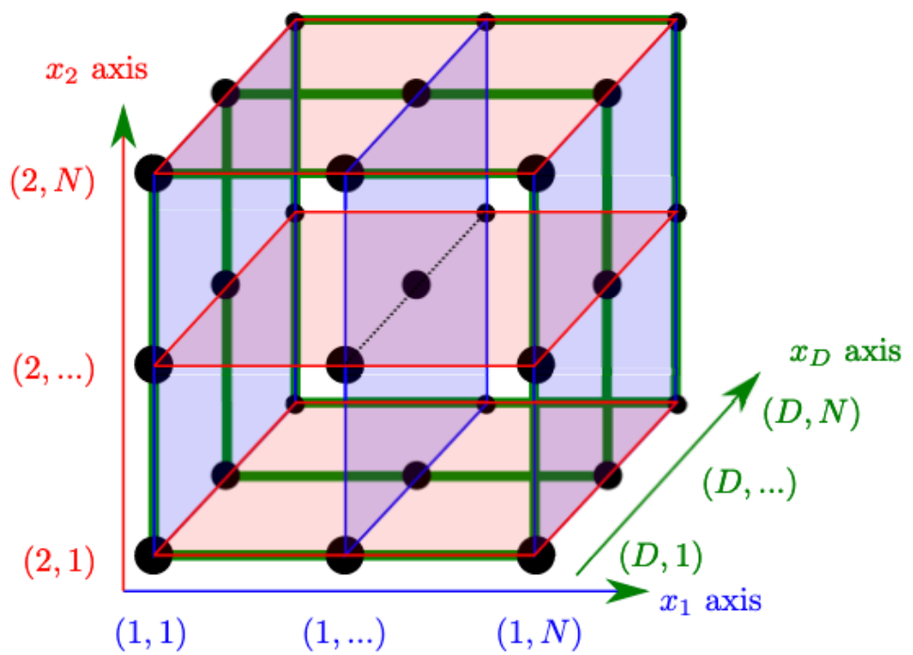
Previous slide: square of side $\sqrt{N}$

The hypercube technique: hypercube of dimension $\log_2 N$ (each side has a size of 2)

Emulating $\log_2 N$ subprotocols with 2 parties.

Source: Figure from [AGHHJY23]



The $D \times N$ main party slices

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

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Soundness error:

$$\left(\frac{1}{2}\right)^{\log_2 N} = \frac{1}{N}$$

Emulation cost:

$2 \cdot \log_2 N$ parties

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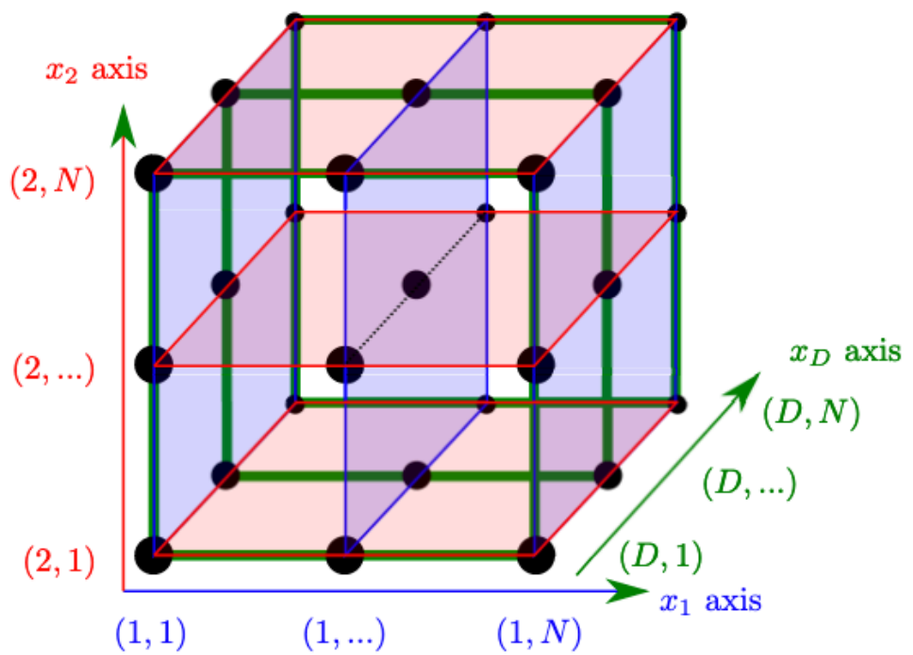

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$1 + \log_2 N$ parties

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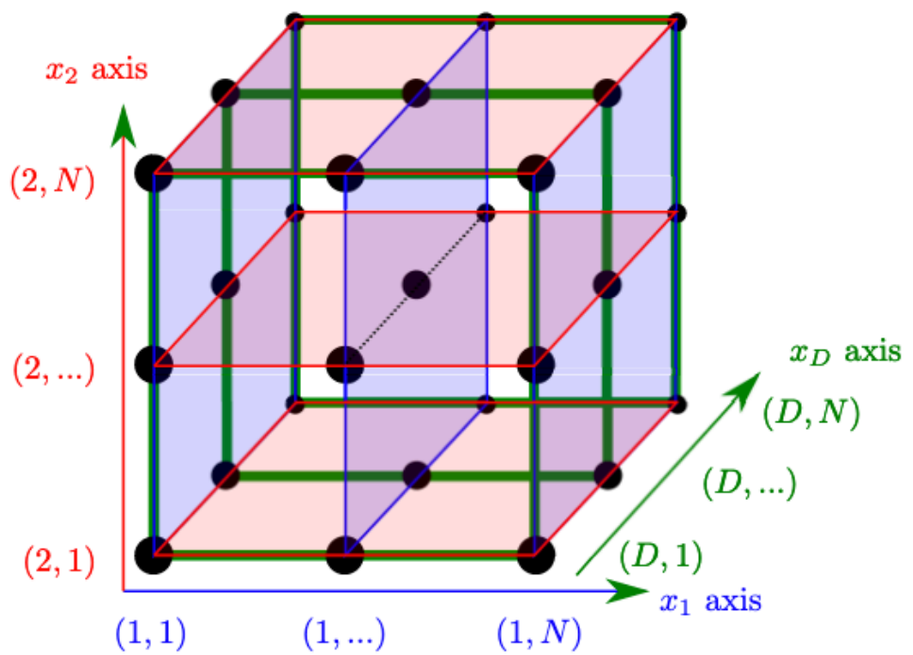


The $D \times N$ main party slices

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

Traditional: N party emulations per repetition

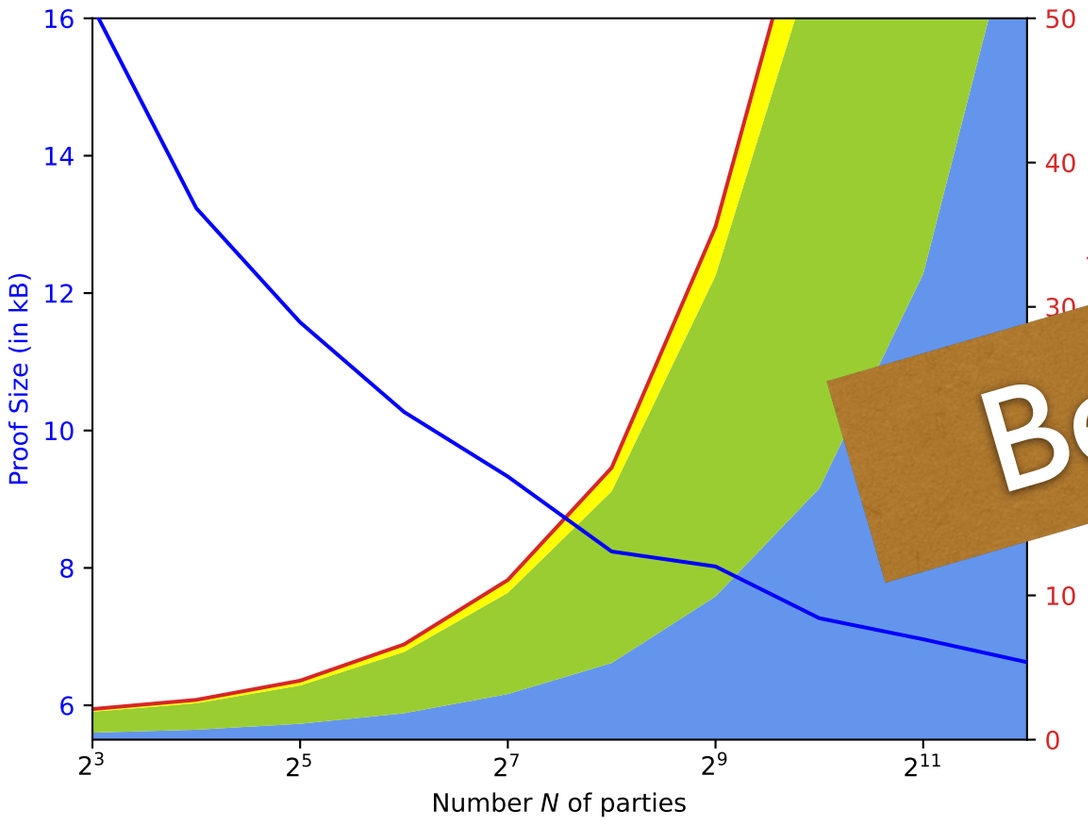
$$N = 256$$

Hypercube: $1 + \log_2 N$ party emulations per repetition

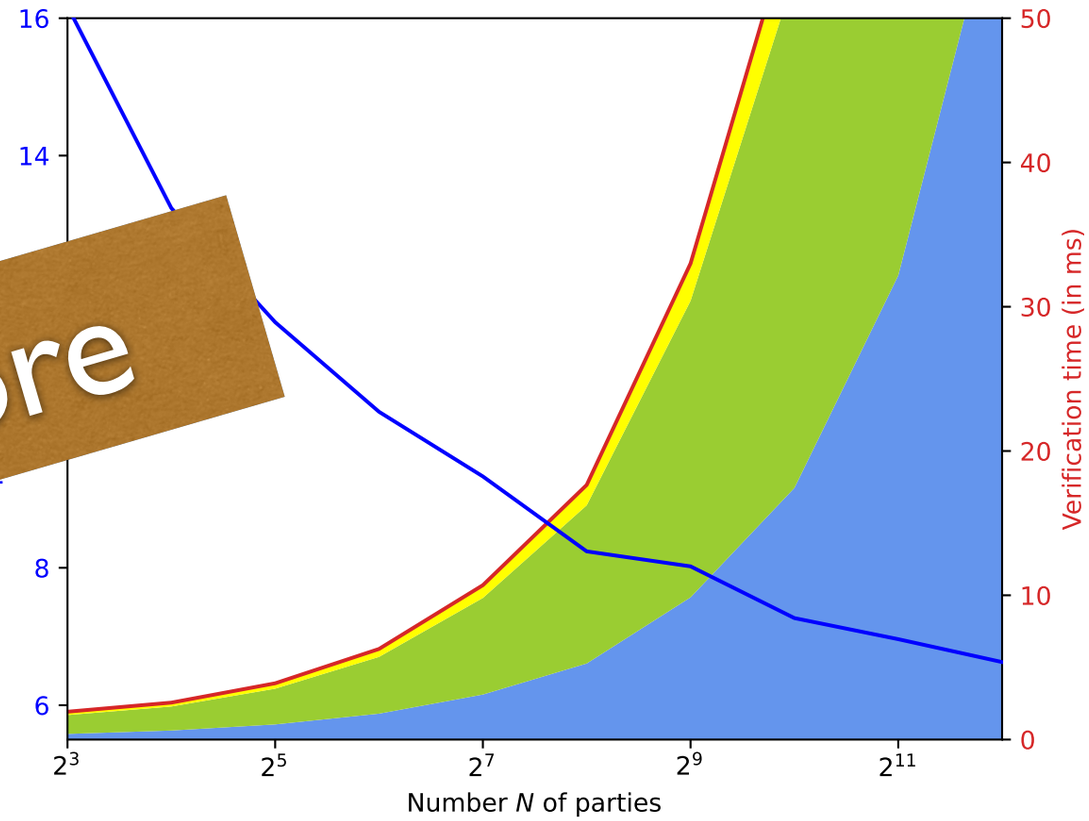
$$1 + \log_2 N = 9$$

The Hypercube Technique

Signing algorithm



Verification algorithm

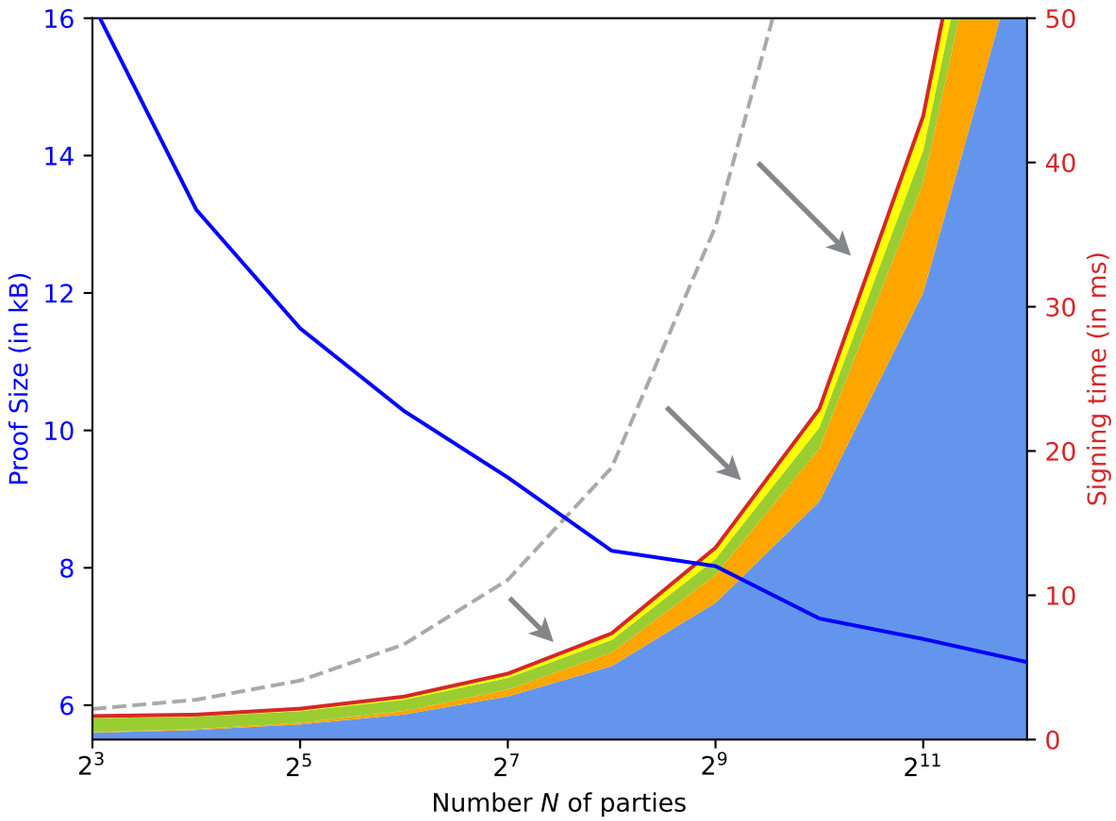


Before

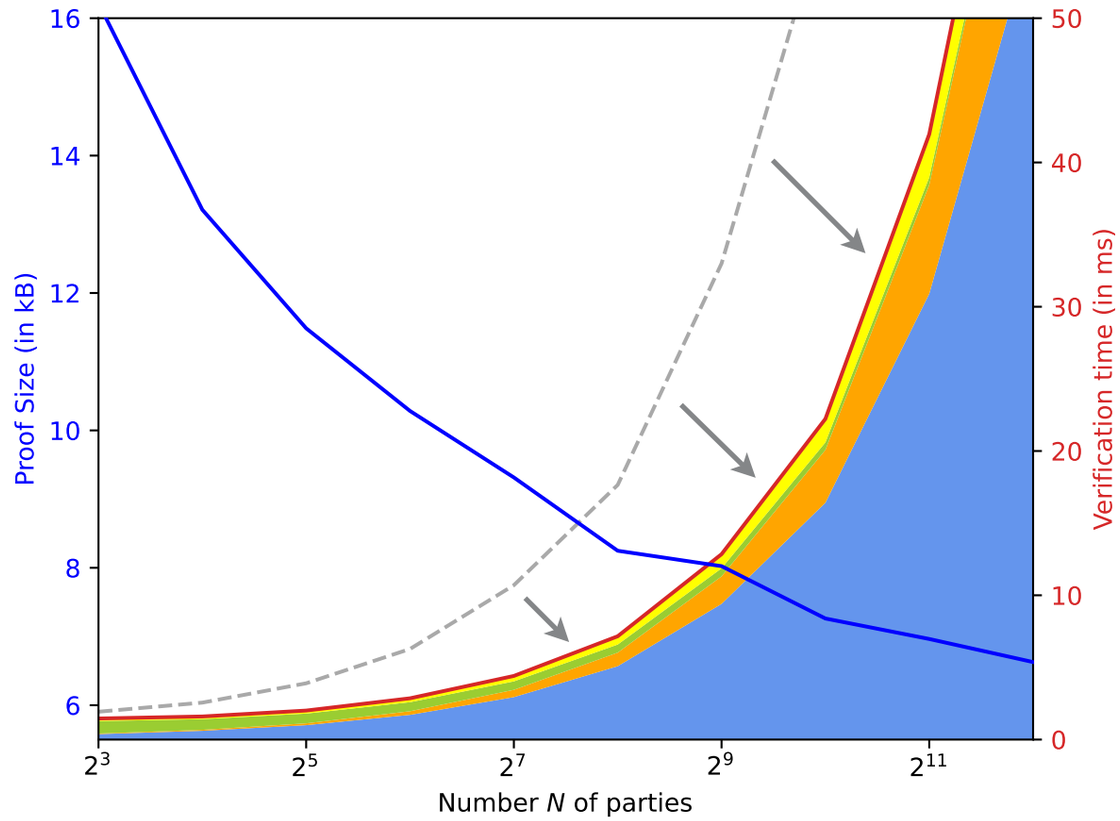
Running times @3.80Ghz

The Hypercube Technique

Signing algorithm



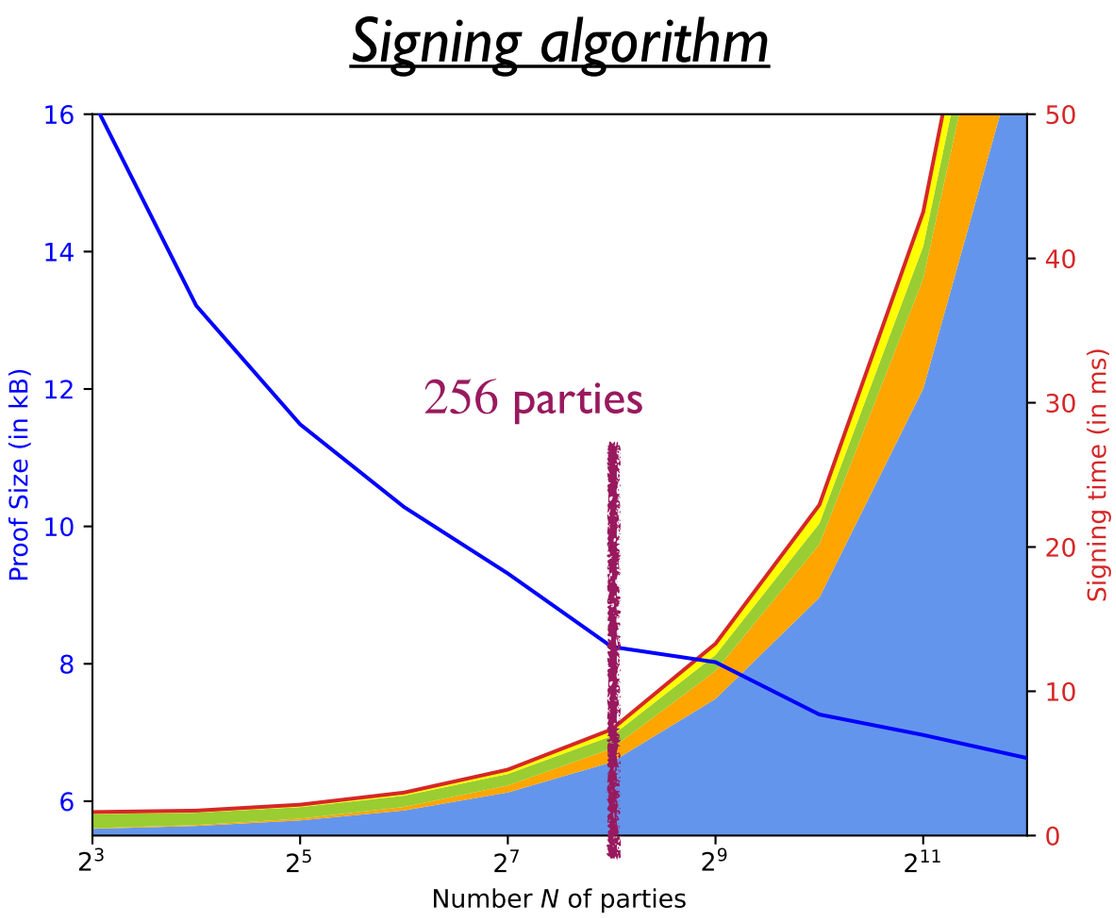
Verification algorithm

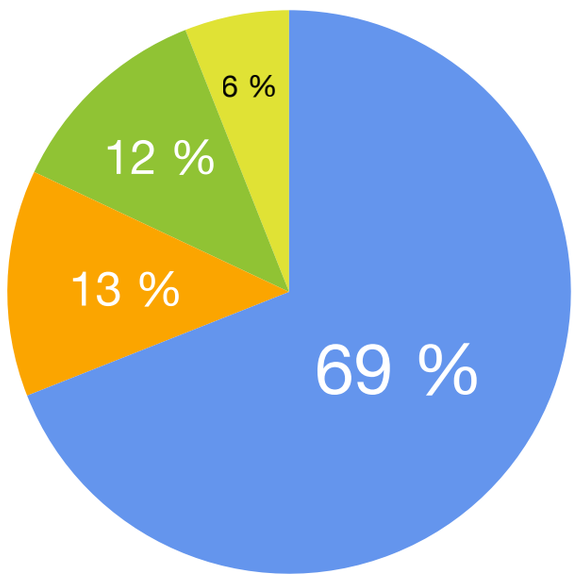


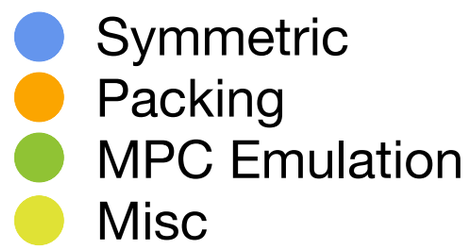
- Symmetric
- Packing
- MPC Emulation
- Misc

Running times @3.80Ghz

The Hypercube Technique







Signing time
for $N := 256$ parties
(7 ms)

Running times @3.80Ghz

The Threshold Approach

[FR22] Feneuil, Rivain: "Threshold Linear Secret Sharing to the Rescue of MPC-in-the-Head" (ePrint 2022/1407)

In the *threshold* approach, we used an **low-threshold** sharing scheme.
For example, the Shamir's $(\ell + 1, N)$-secret sharing scheme.

To share a value x,

- sample $r_1, r_2, \ldots, r_\ell$ uniformly at random,
- build the polynomial $P(X) = x + \sum_{k=0}^{\ell} r_k \cdot X^k$,
- Set the share $[\![x]\!]_i \leftarrow P(e_i)$, where e_i is publicly known.

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The prover reveals only ℓ shares to the verifier (instead of $N - 1$).

In practice, $\ell \in \{1,2,3\}$.

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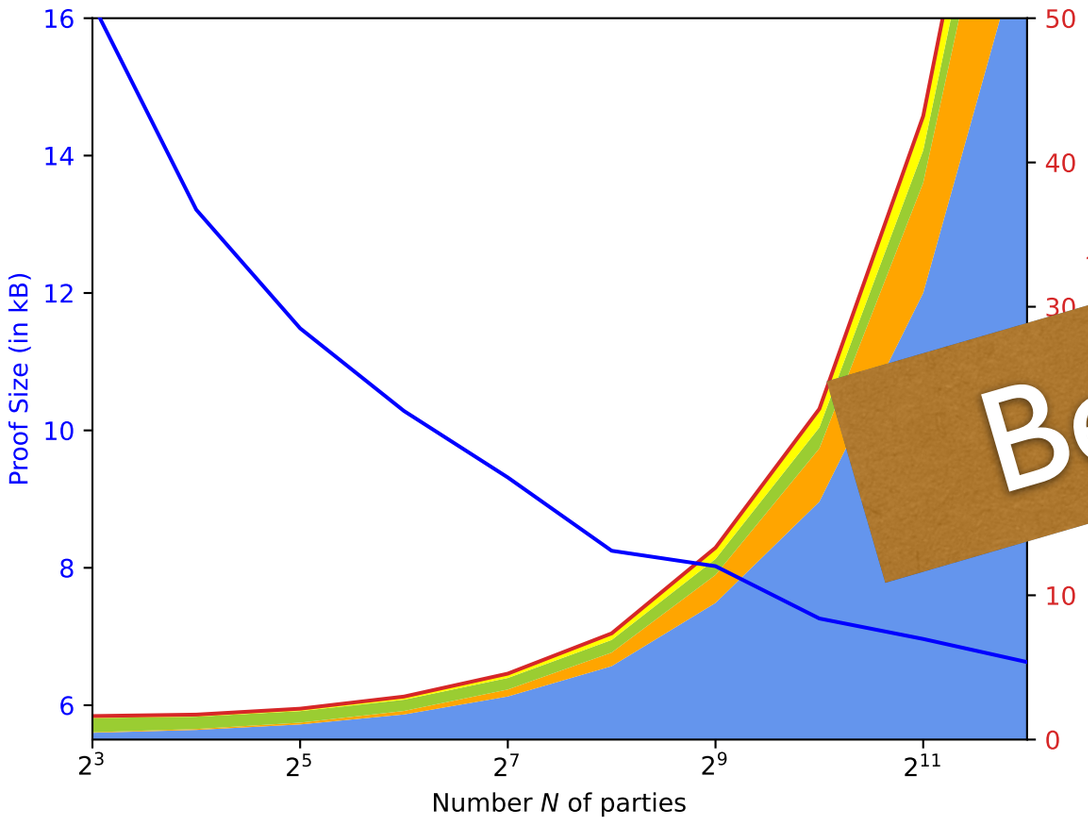
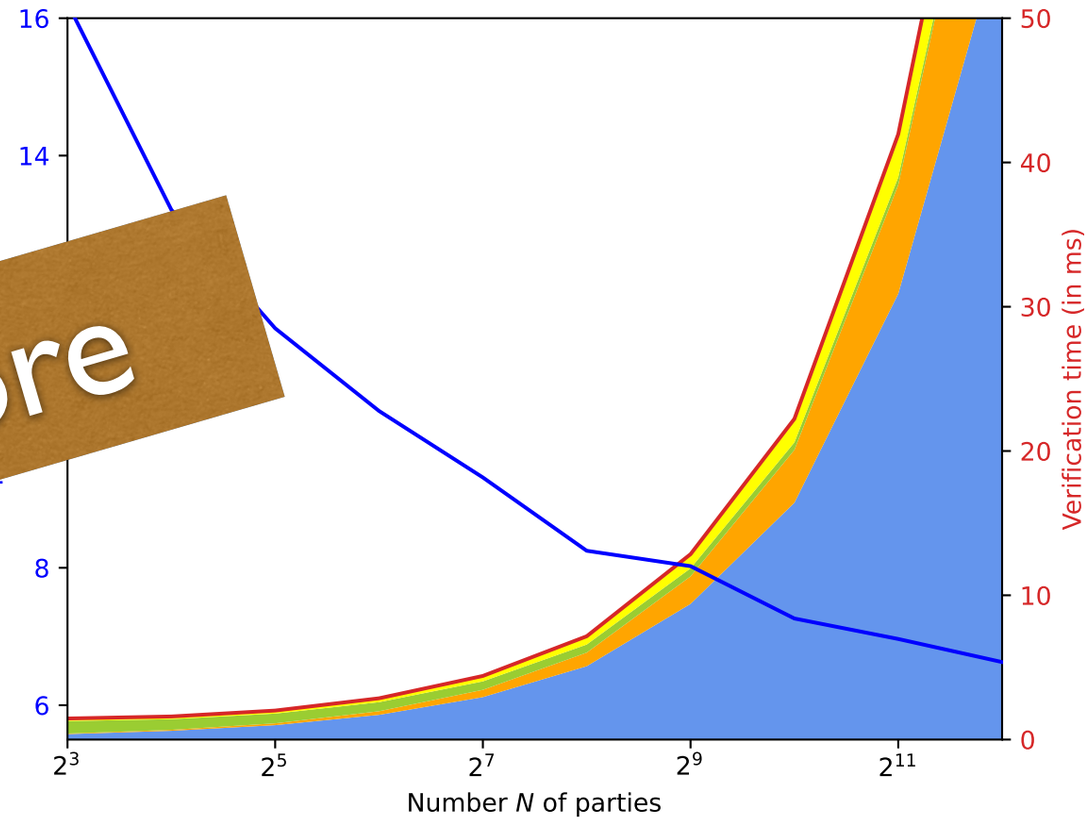
- The prover uses a Merkle tree to commit the shares;
- The obtained signature size is **larger**;
- We have the constraint: $N \leq |\mathbb{F}|$.

The Threshold Approach

Signing algorithm



Verification algorithm

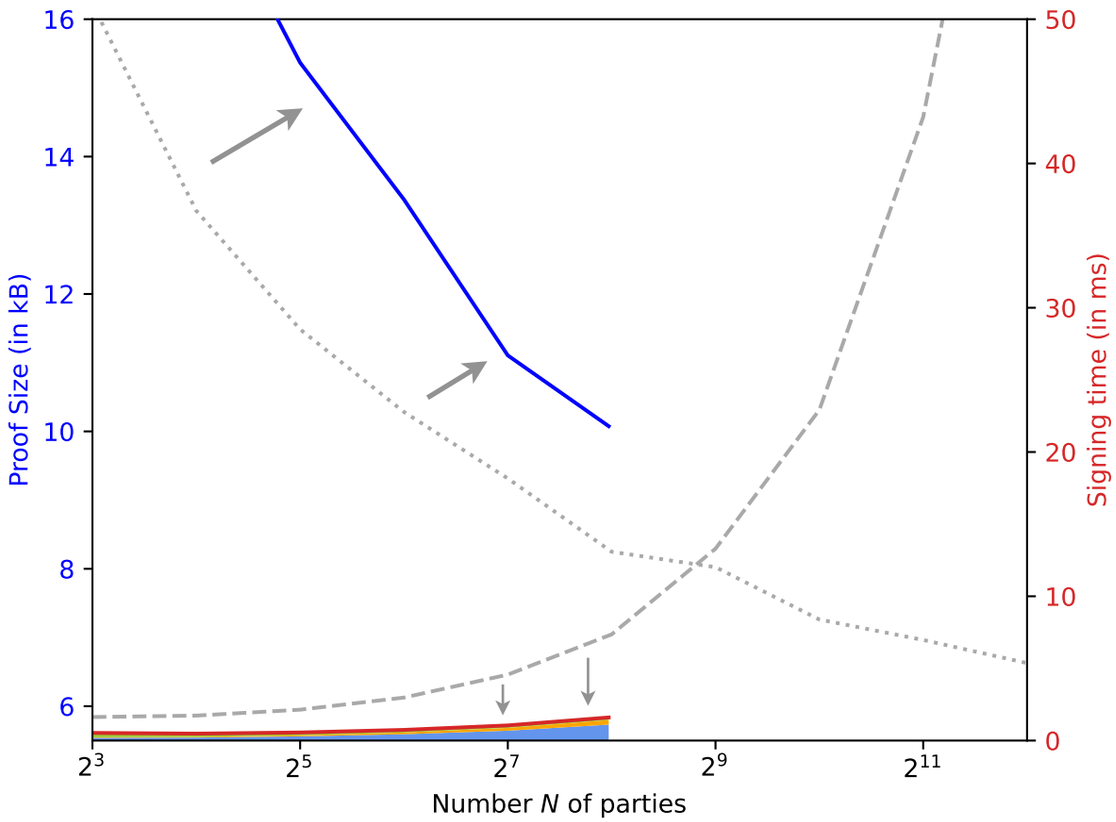


Before

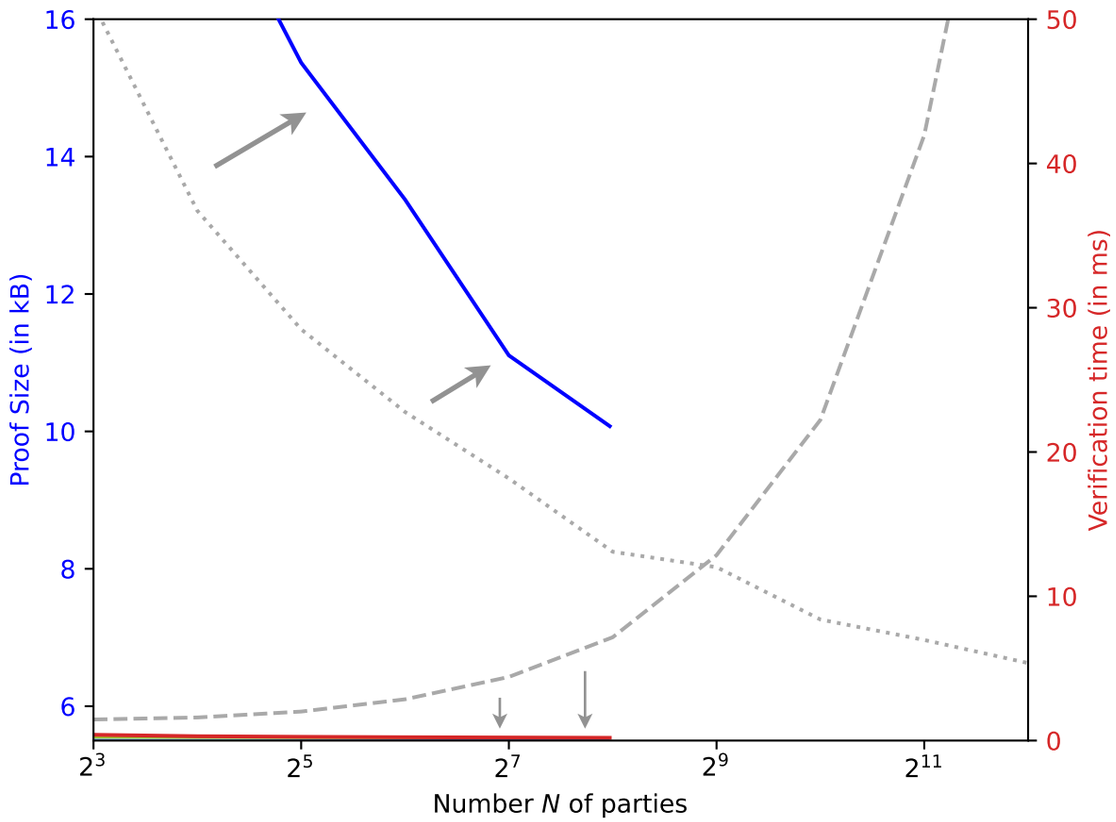
Running times @3.80Ghz

The Threshold Approach

Signing algorithm

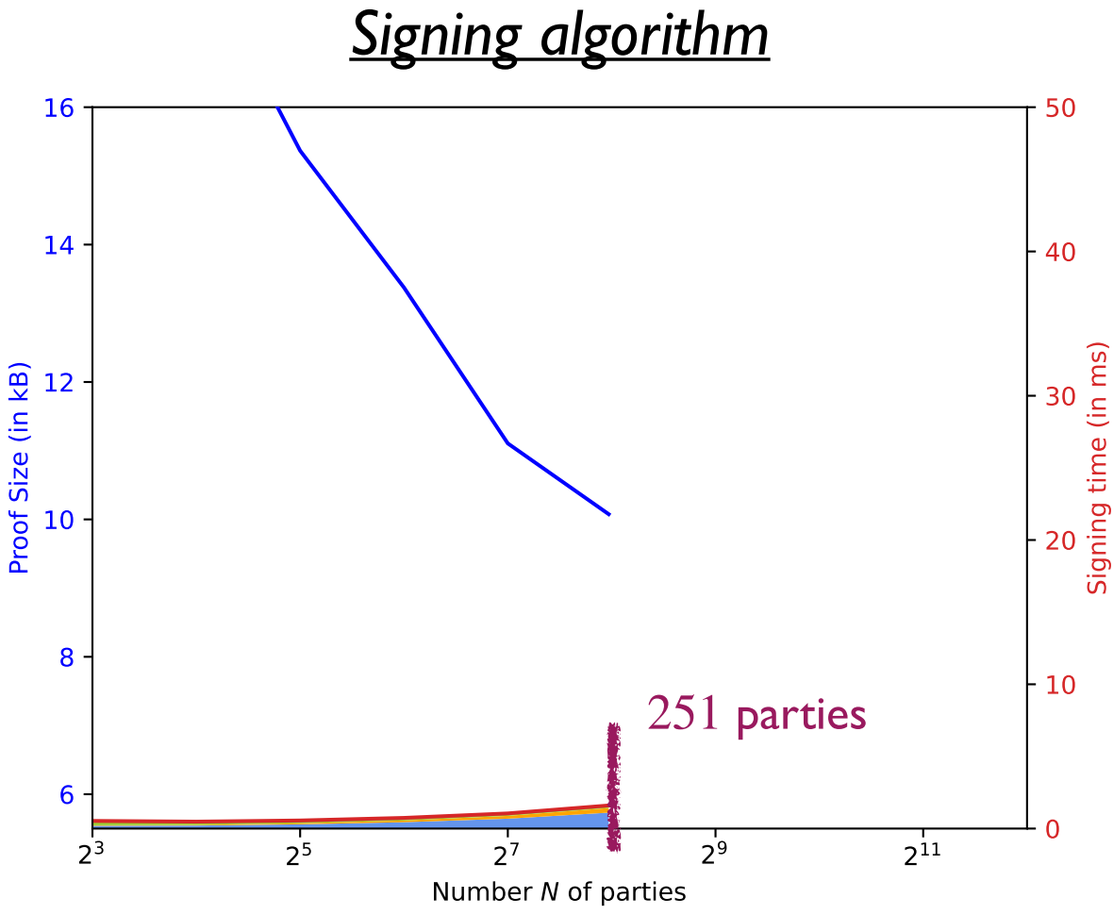


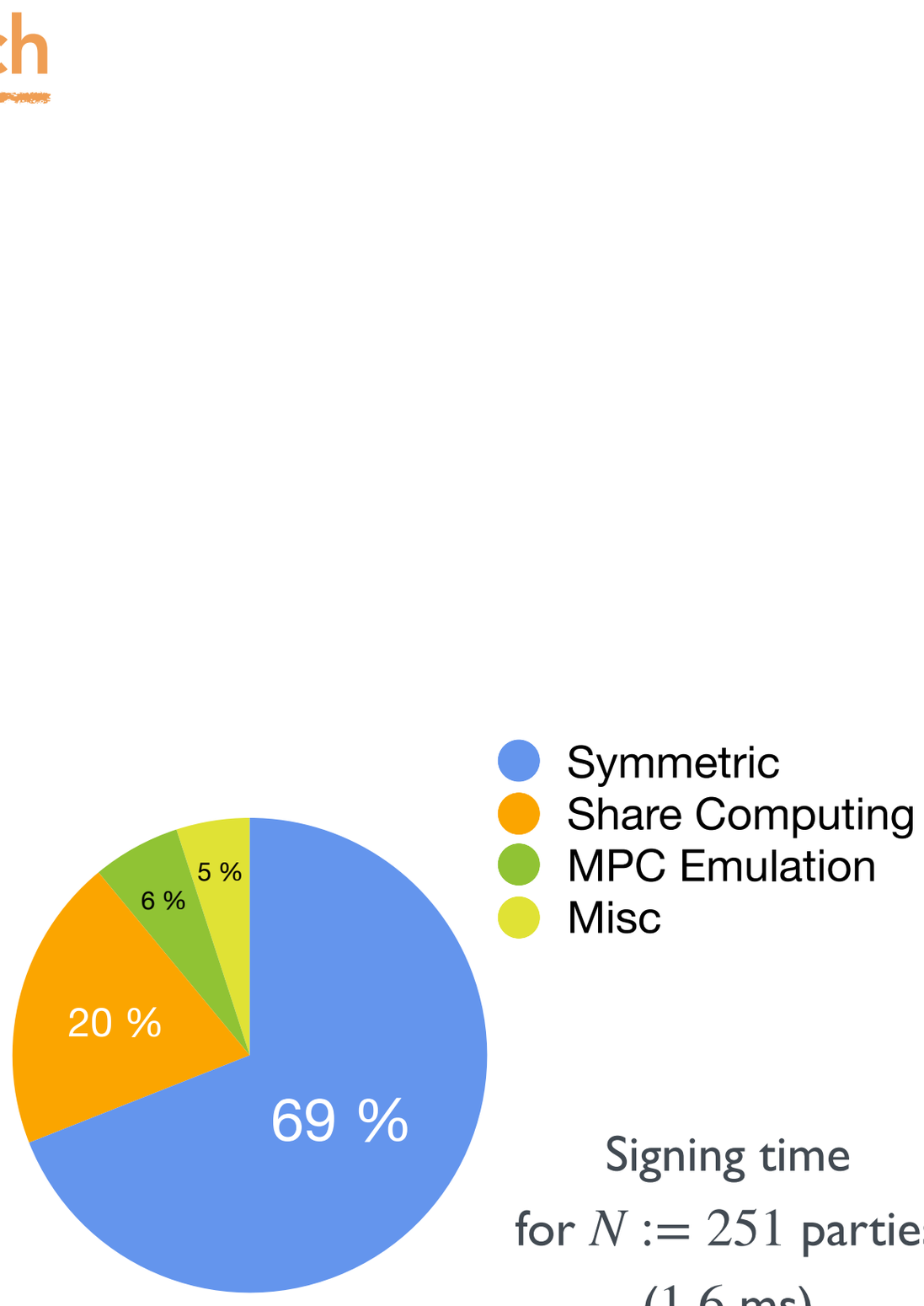
Verification algorithm



Running times @3.80Ghz

The Threshold Approach

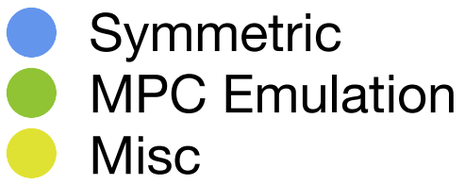


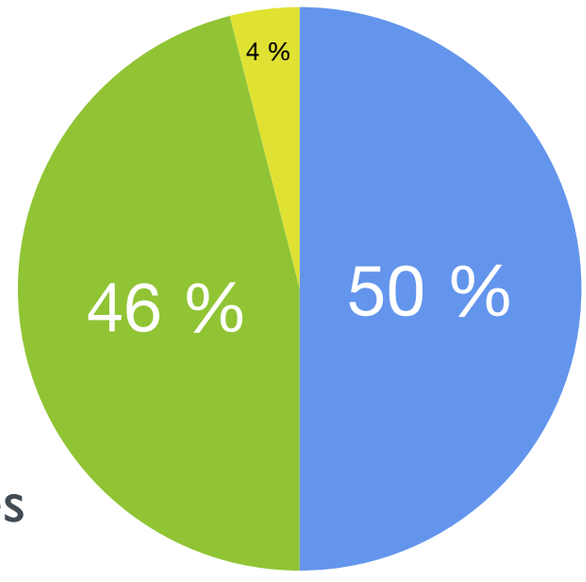


Running times @3.80Ghz

Signing time
for $N := 251$ parties
(1.6 ms)

The Threshold Approach

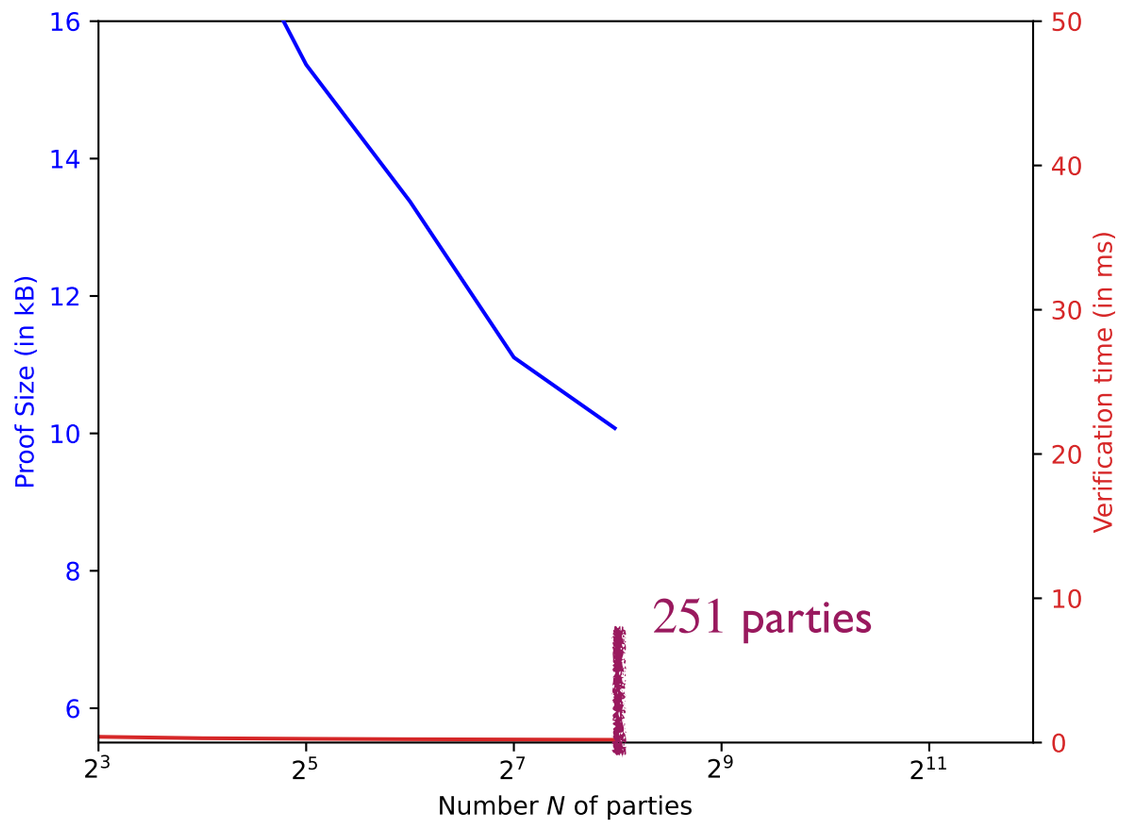




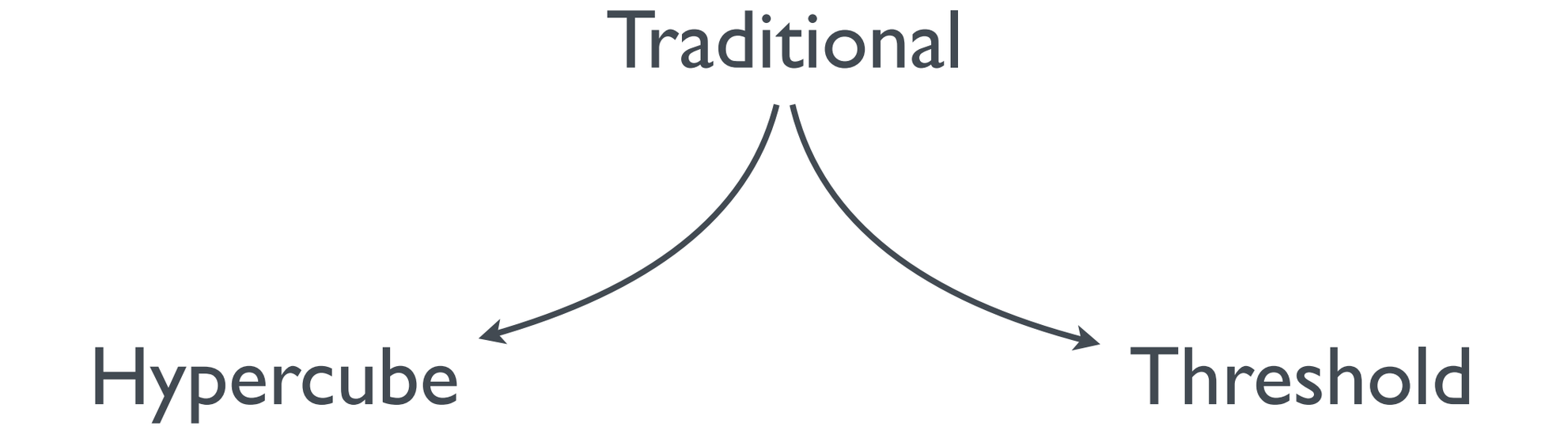
Verification time
for $N := 251$ parties
(0.2 ms)

Running times @3.80Ghz

Verification algorithm



The existing MPCitH transforms



Shorter signature sizes
Highly parallelizable
Slower signing time
Signing time ≈ Verification time
Computational cost is mainly
due to symmetric primitives

Faster signing time
Highly parallelizable
Very fast verification
Larger signature size
Restriction # of parties
Computational cost is mainly
due to arithmetics

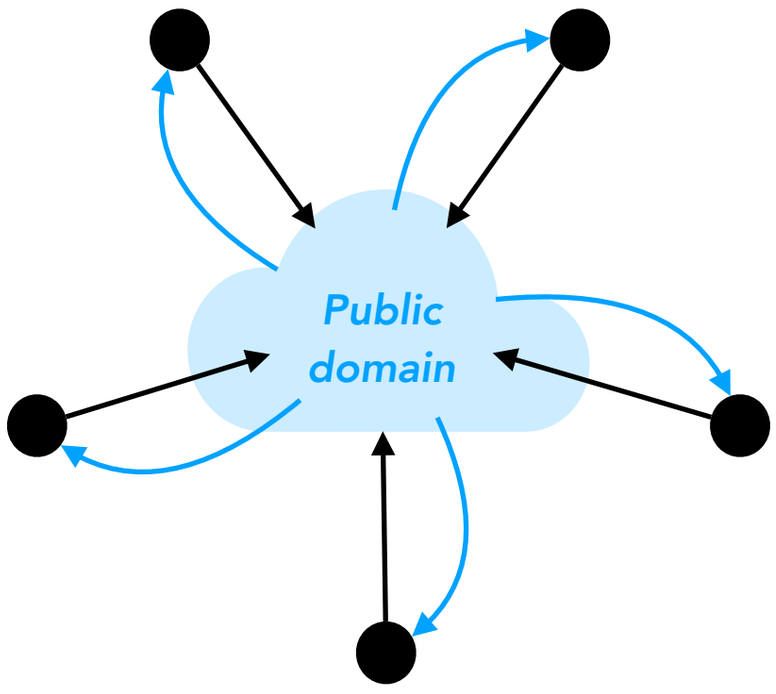
MPCitH-based NIST candidates

	Short Instance	**Fast Instance**
AIMer	Traditional (256-1615)	Traditional (16-57)
Biscuit	Traditional (256)	Traditional (16)
MIRA	Hypercube (256)	Hypercube (32)
MiRith	Traditional (256)	Traditional (16)
	Hypercube (256)	Hypercube (16)
MQOM	Hypercube (256)	Hypercube (32)
RYDE	Hypercube (256)	Hypercube (32)
SDitH	Hypercube (256)	Threshold (251-256)

Related works

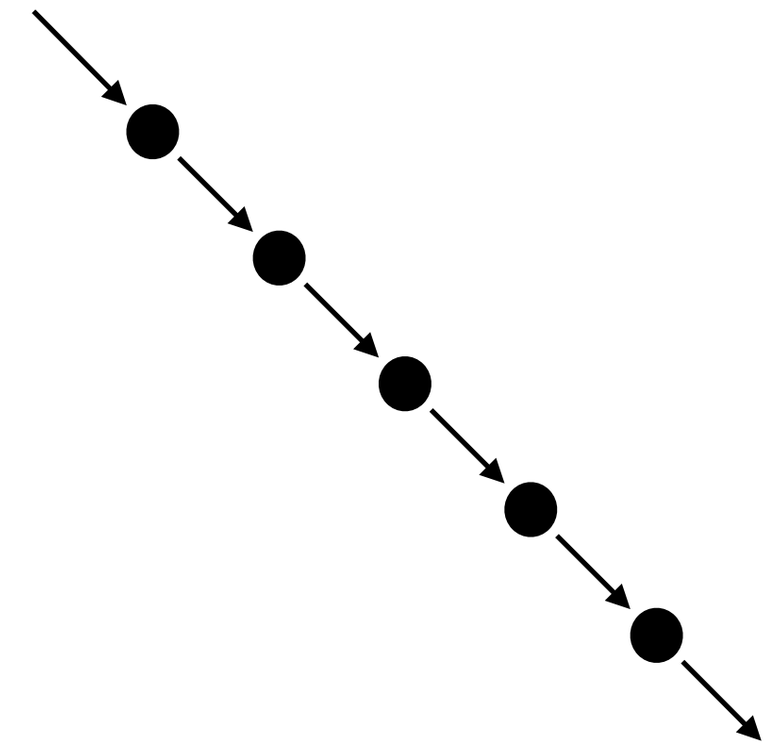
PERK: Shared Permutation on Permuted Kernel Problem

Standard MPC-in-the-Head



AIMer, Biscuit, MIRA, MiRitH
MQOM, RYDE, SDitH

Path-based MPC-in-the-Head



PERK

FAEST: VOLE-in-the-Head

VOLE: *vector oblivious linear evaluation*

"FAEST is the first AES-based signature scheme to be smaller than SPHINCS+"

Will be presented at Crypto'23 the 23rd August

Conclusion

Advantages and limitations

- Limitations
  - Relatively ***slow*** *(few milliseconds)*
    - Greedy use of symmetric cryptography
  - Relatively ***large*** signatures (*4-10 KB for L1*)
  - Signature size: ***quadratic*** growth in the security level

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- Advantages
  - ***Conservative*** hardness assumption:
    - No structure (often), no trapdoor
  - ***Small*** (public) keys
  - ***Good*** public key + signature size
  - Adaptive and ***tunable*** parameters

Conclusion

- MPC-in-the-Head
  - Very versatile and tunable
  - Can be applied on any one-way function
  - A practical tool to build *conservative* signature schemes

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    Could lead to follow-up works

  - Signatures with advanced functionalities:

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Thank you for your attention.