



Post-quantum Signatures from Secure Multiparty Computation

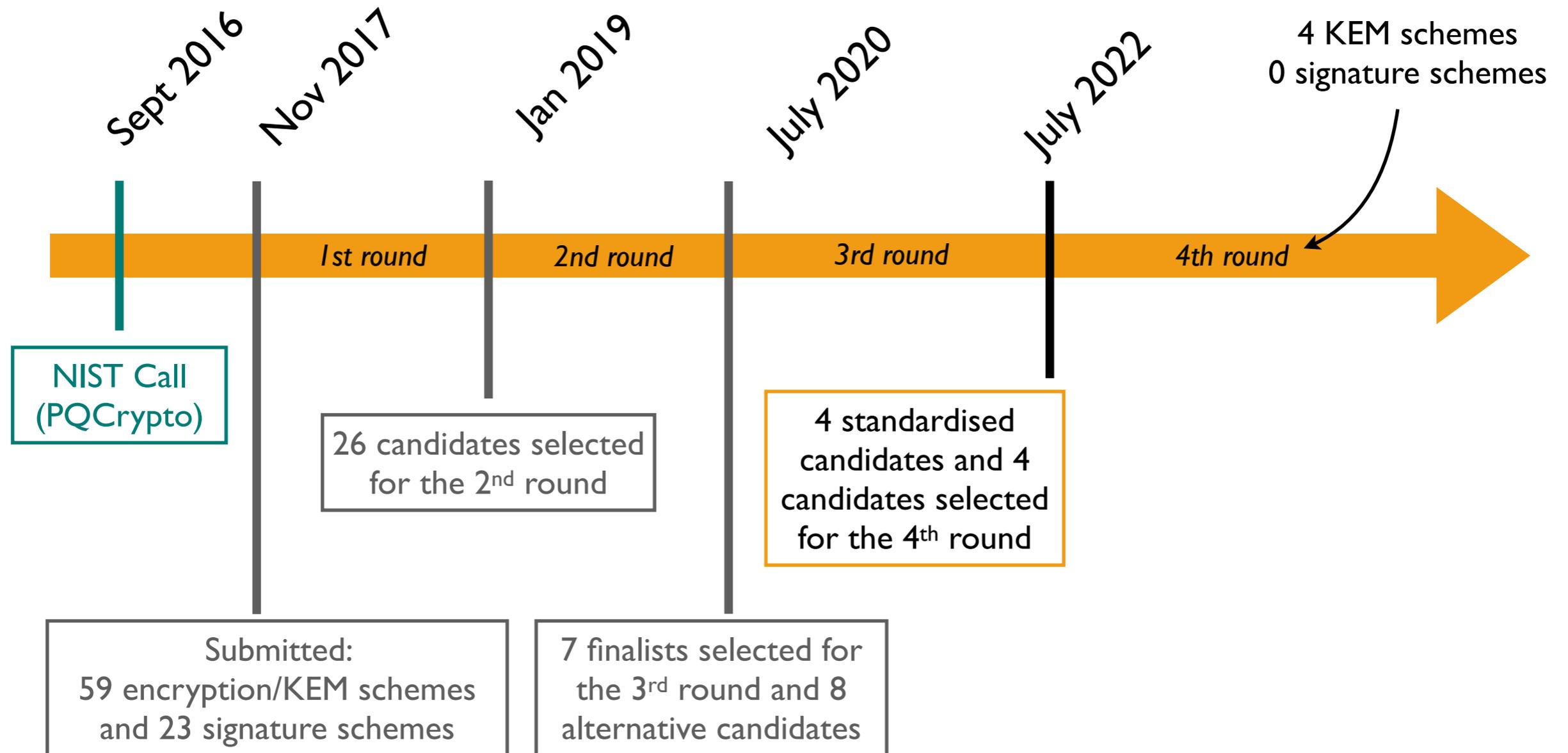
Thibauld Feneuil



June 14th, 2023 — WRACH'23

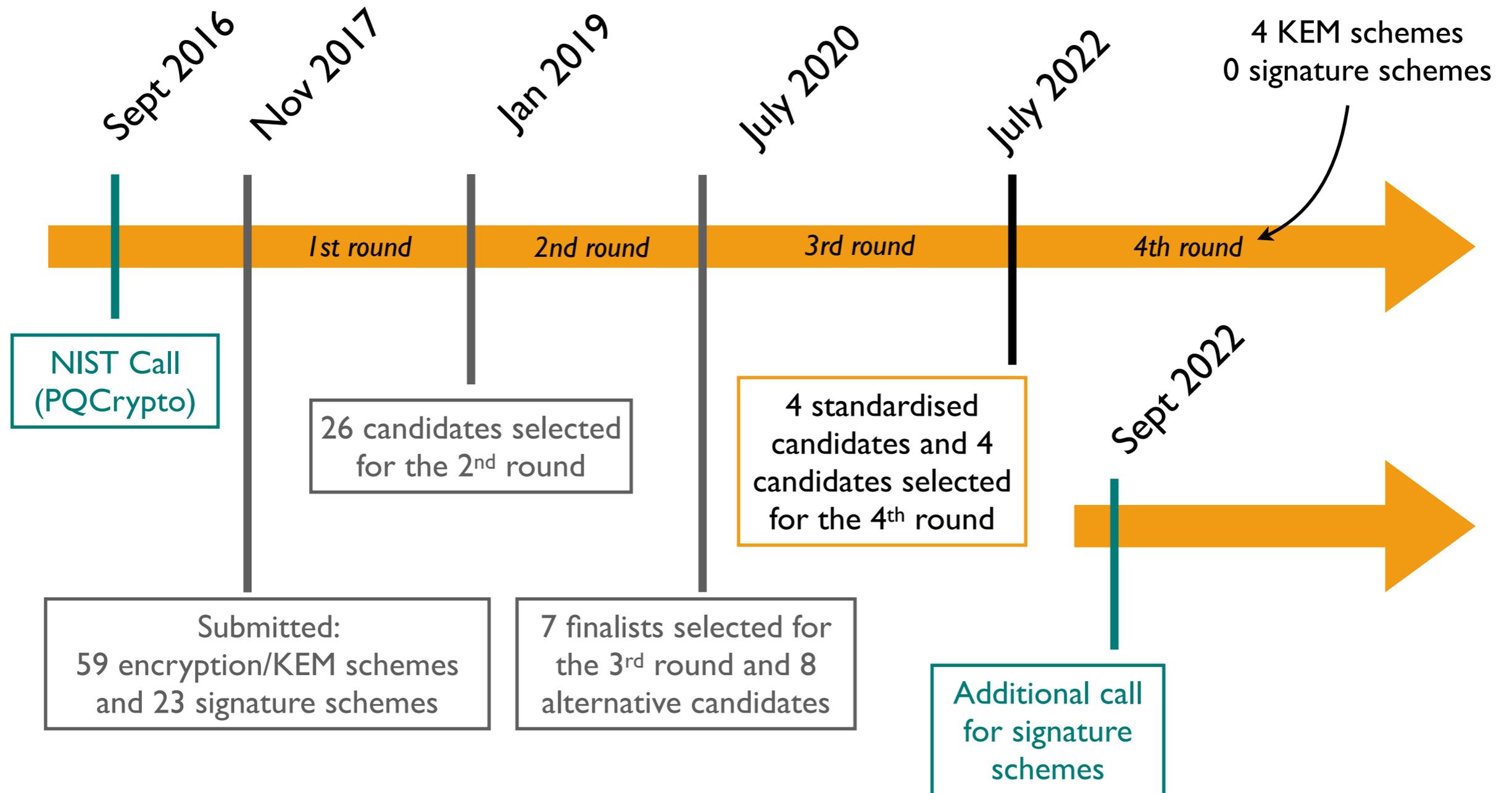
Context

- Additional NIST call for quantum-resilient signature schemes



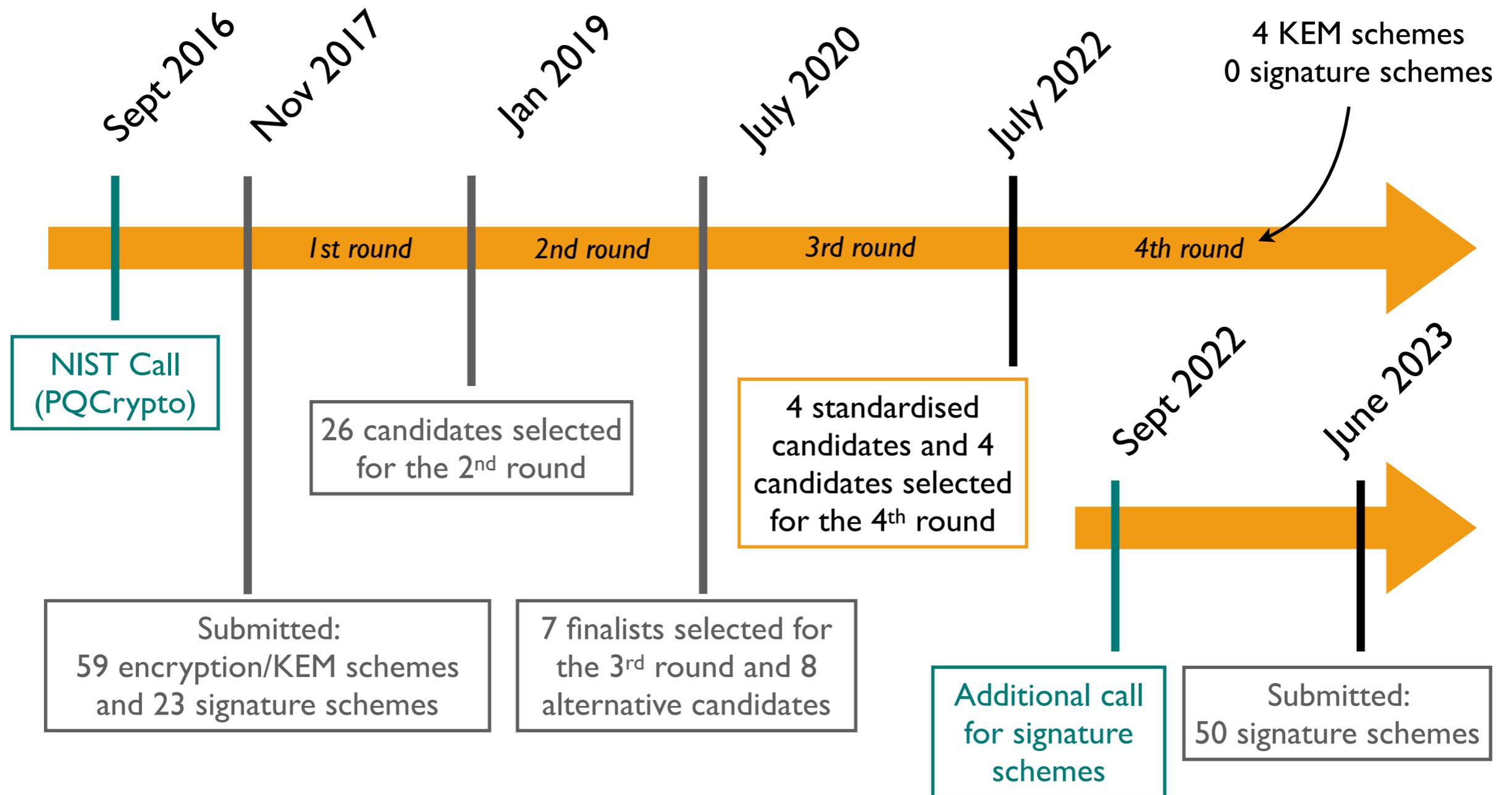
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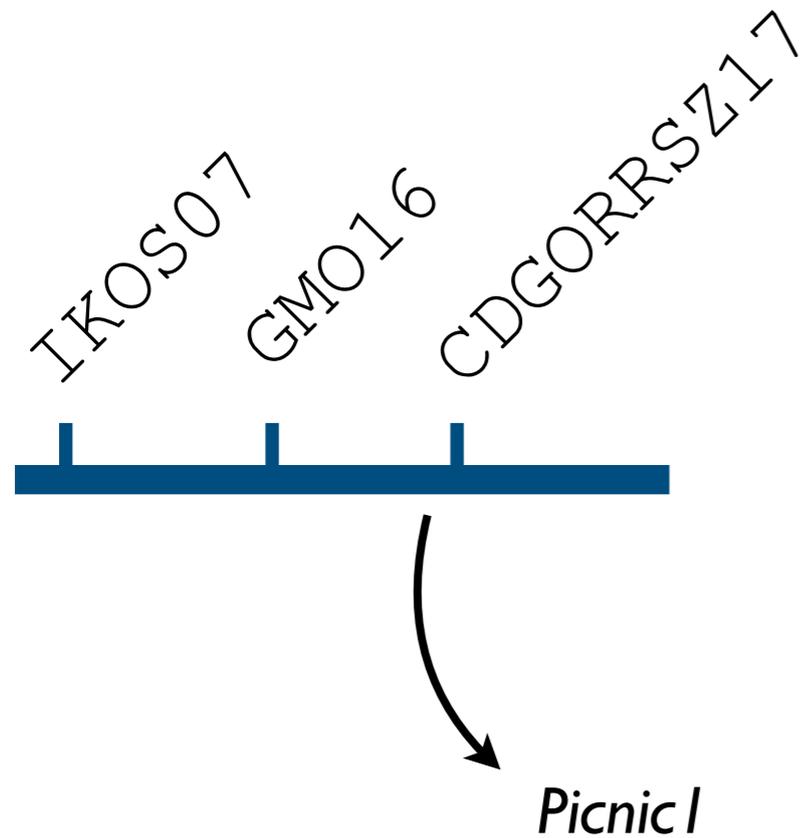
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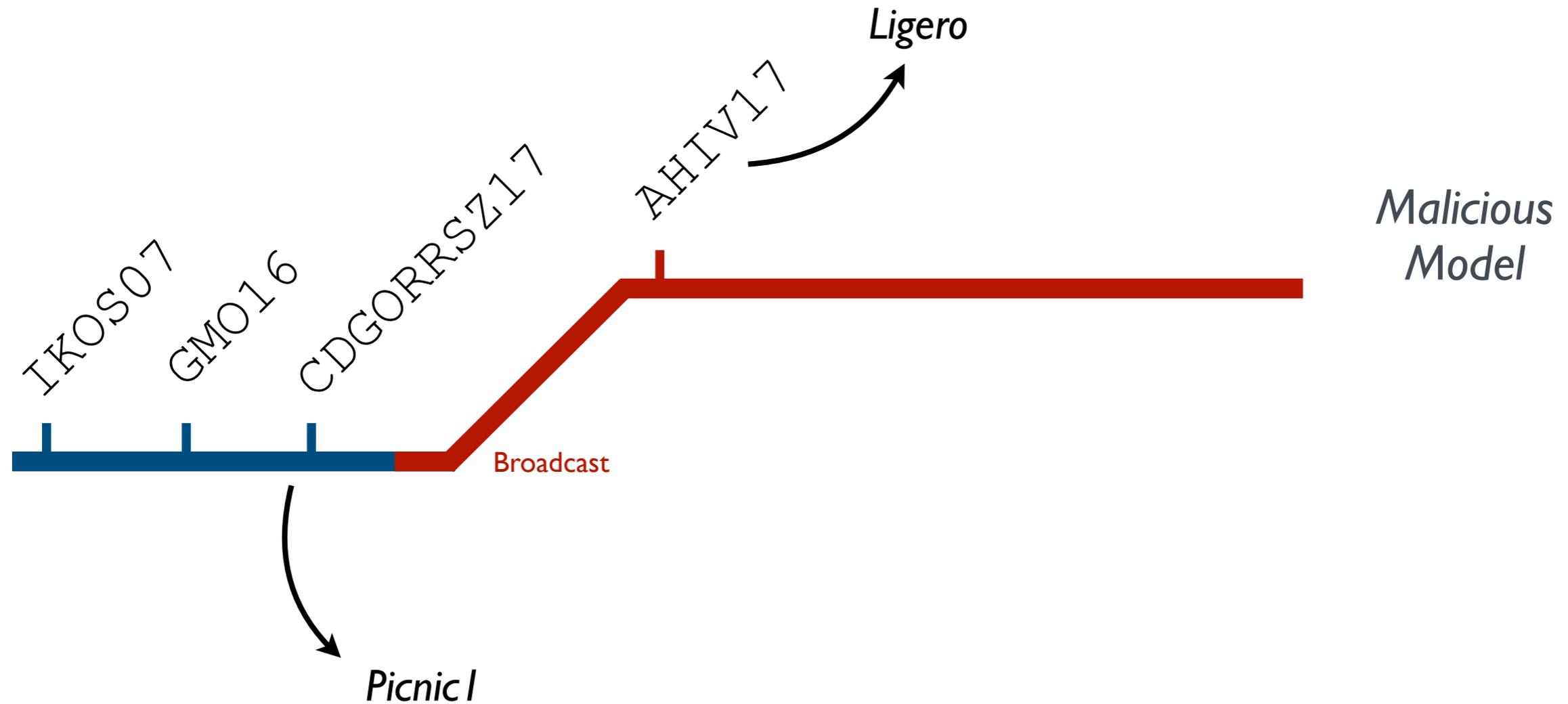
Type	Number
Lattice	8
Code-based	5
Multivariate	11
MPC in the head	7
Symmetric	6
Isogeny	1
Other	12
Total	50

Source: NIST, 9th June 2023

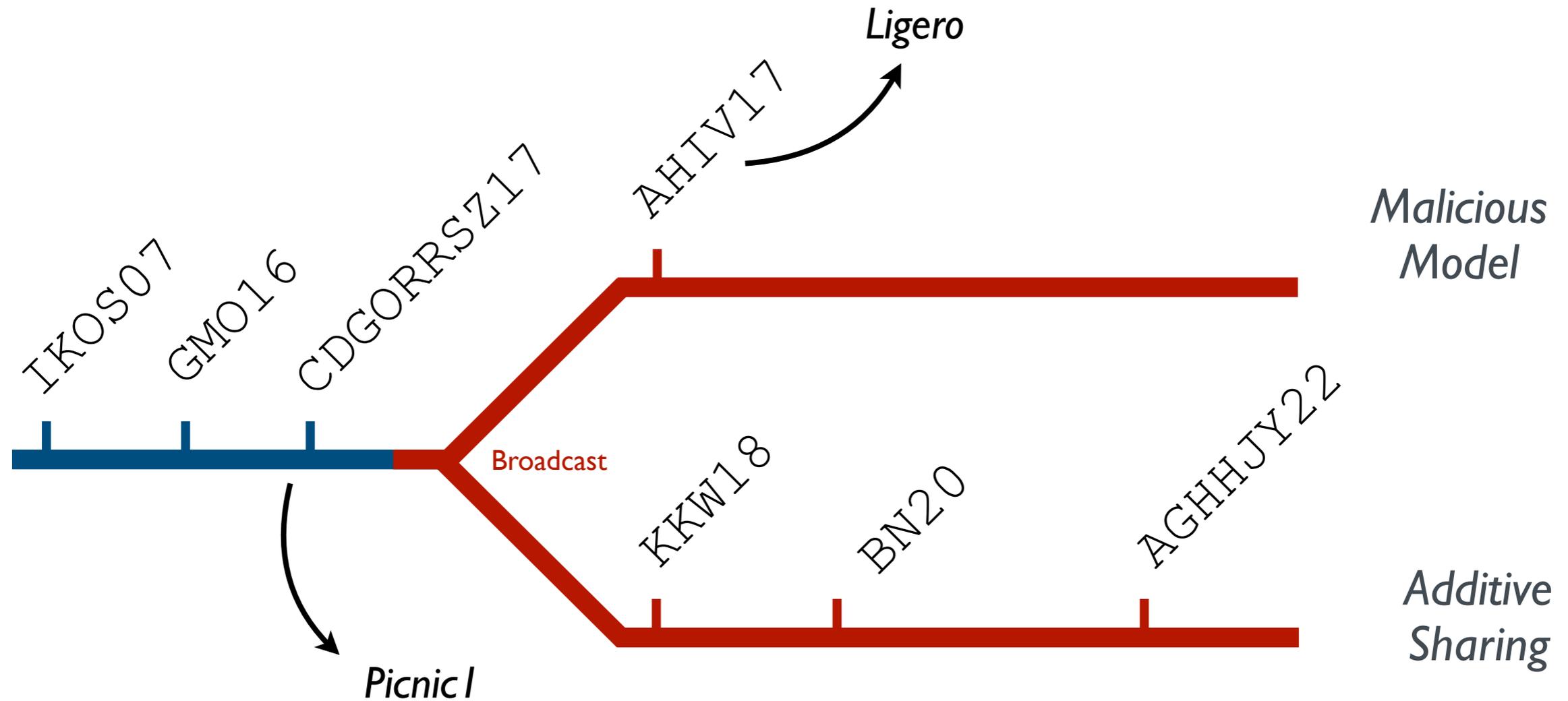
MPC-in-the-Head Paradigm



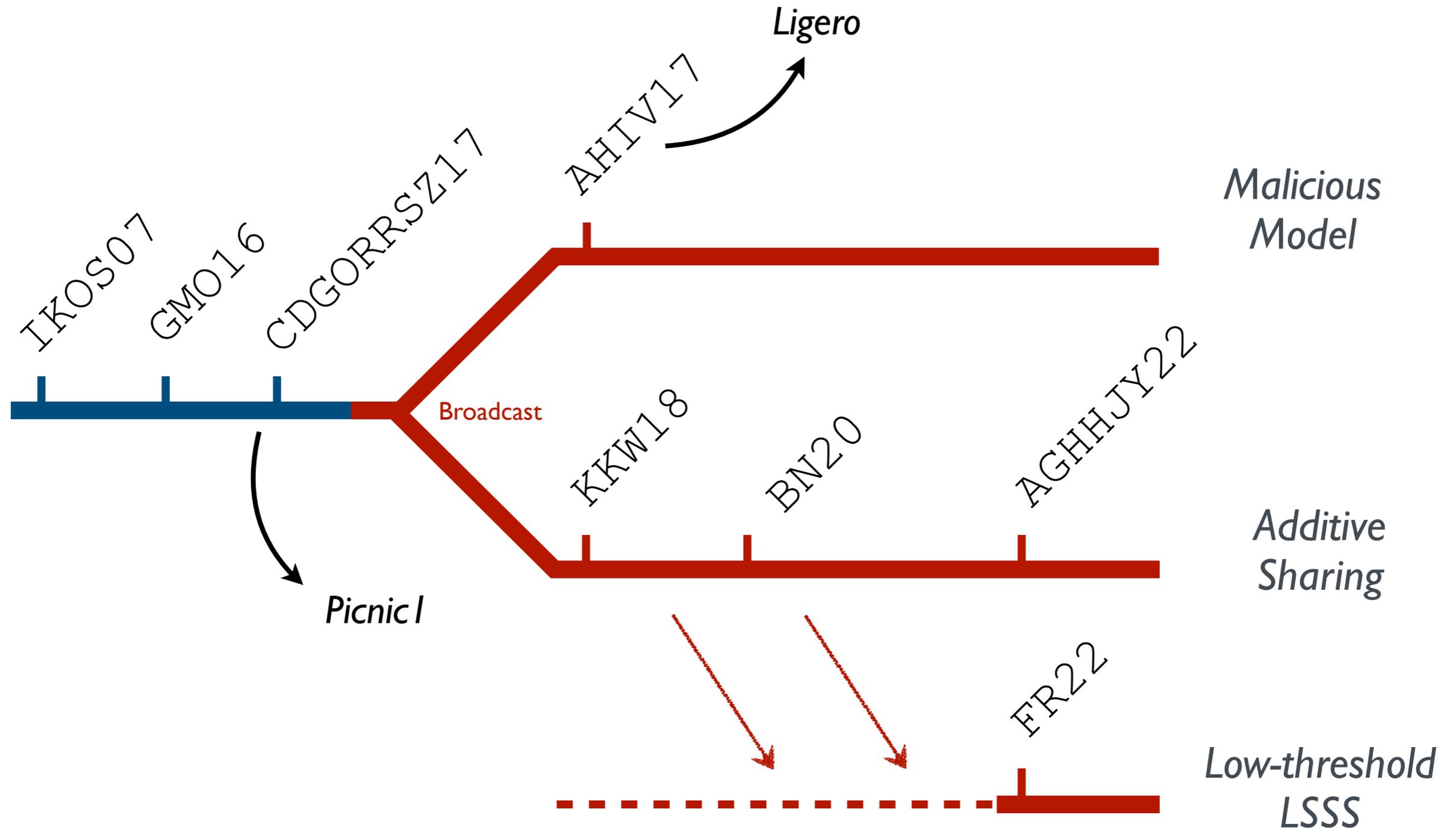
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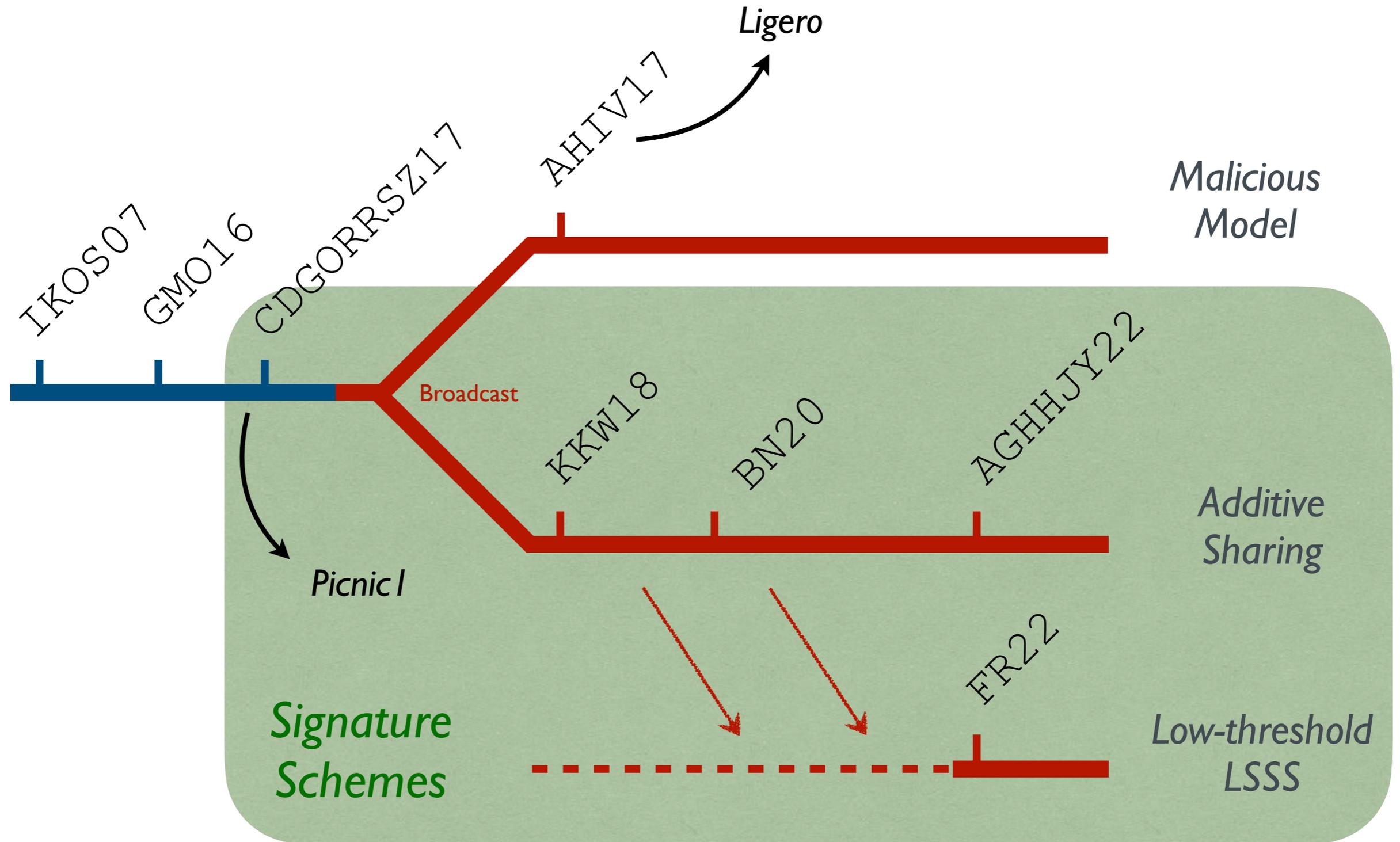
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MPC-in-the-Head Paradigm



Build a signature scheme from MPC

- Choose an one-way function F .

Methodology

Build a signature scheme from MPC

[FJR22]

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- Syndrome decoding problem: given (H, y) , find a vector x such that $y = Hx$ and $w_H(x) \leq w$.

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$$Q(X) \cdot \left(\sum_{i=1}^m x_i \prod_{j=1, j \neq i}^m \frac{X - \gamma_j}{\gamma_i - \gamma_j} \right) = P(X) \cdot \left(\prod_{i=1}^m (X - \gamma_i) \right)$$
with $\deg Q = w$.

Rephrase the pre-image verification

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Can be zero for at most w values

Must be zero for at least w coordinates
($w_H(x) \leq x$)

Rephrase the pre-image verification

To get a valid polynomial Q , we can take

$$Q(X) := Q'(X) \cdot \prod_{i=1, x_i \neq 0}^m (X - \gamma_i)$$

$Q(\gamma_k) \cdot x_k = 0$

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- Design a dedicated MPC protocol for the pre-image verification.

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Design a MPC protocol for SD

We need to check that the secret x satisfies

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Linear relation
⇒ Easy to compute in MPC

The MPC protocol will sample a random public point r and evaluate the polynomial relation on this point.

Schwartz-Zippel Lemma:
If the polynomial relation is not satisfied, then the probability that it is true for a random point is small.

Finally, the MPC protocol just needs to check a quadratic term:
 $Q(r) \cdot S(r) = P(r) \cdot F(r)$

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with $\deg Q = w$.
- An MPC protocol which evaluates the above polynomial relation on a random point (Schwartz-Zippel).

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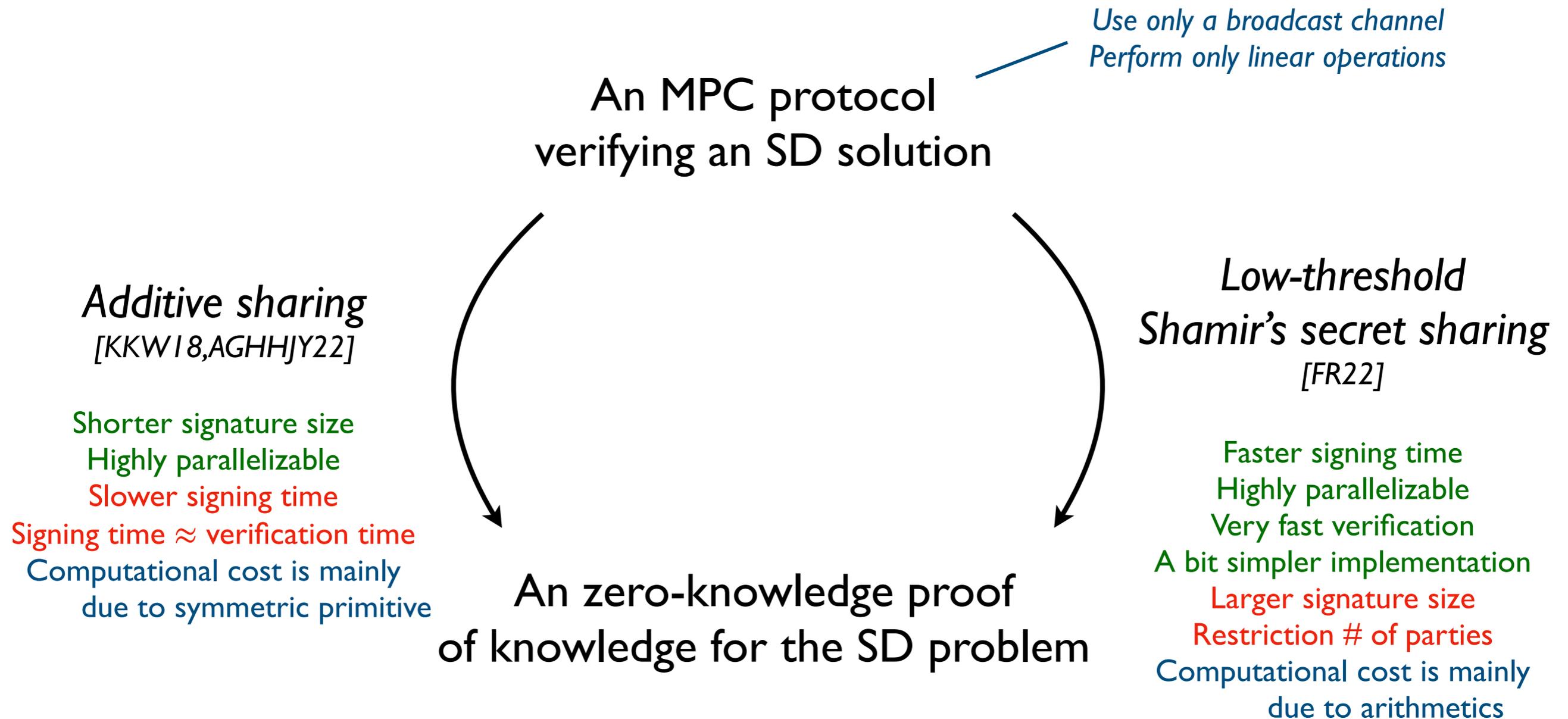
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- Apply a MPC-in-the-Head transformation

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- *Result*: a zero-knowledge proof of knowledge for the syndrome decoding problem.

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- Apply a MPC-in-the-Head transformation
- Make the scheme non-interactive (Fiat-Shamir transformation)

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- An MPC protocol which evaluates the above polynomial relation on a random point (Schwartz-Zippel).
- *Result*: a zero-knowledge proof of knowledge for the syndrome decoding problem.
- *Result*: a signature scheme relying on the syndrome decoding problem.

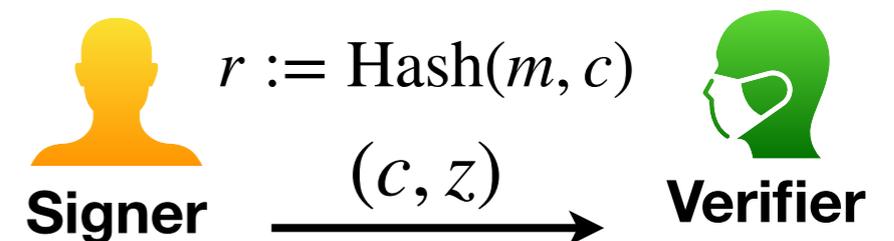
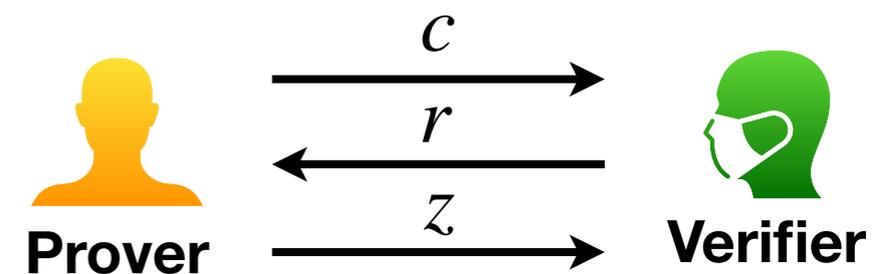
Build a signature scheme from MPC

An zero-knowledge proof of knowledge for the SD problem

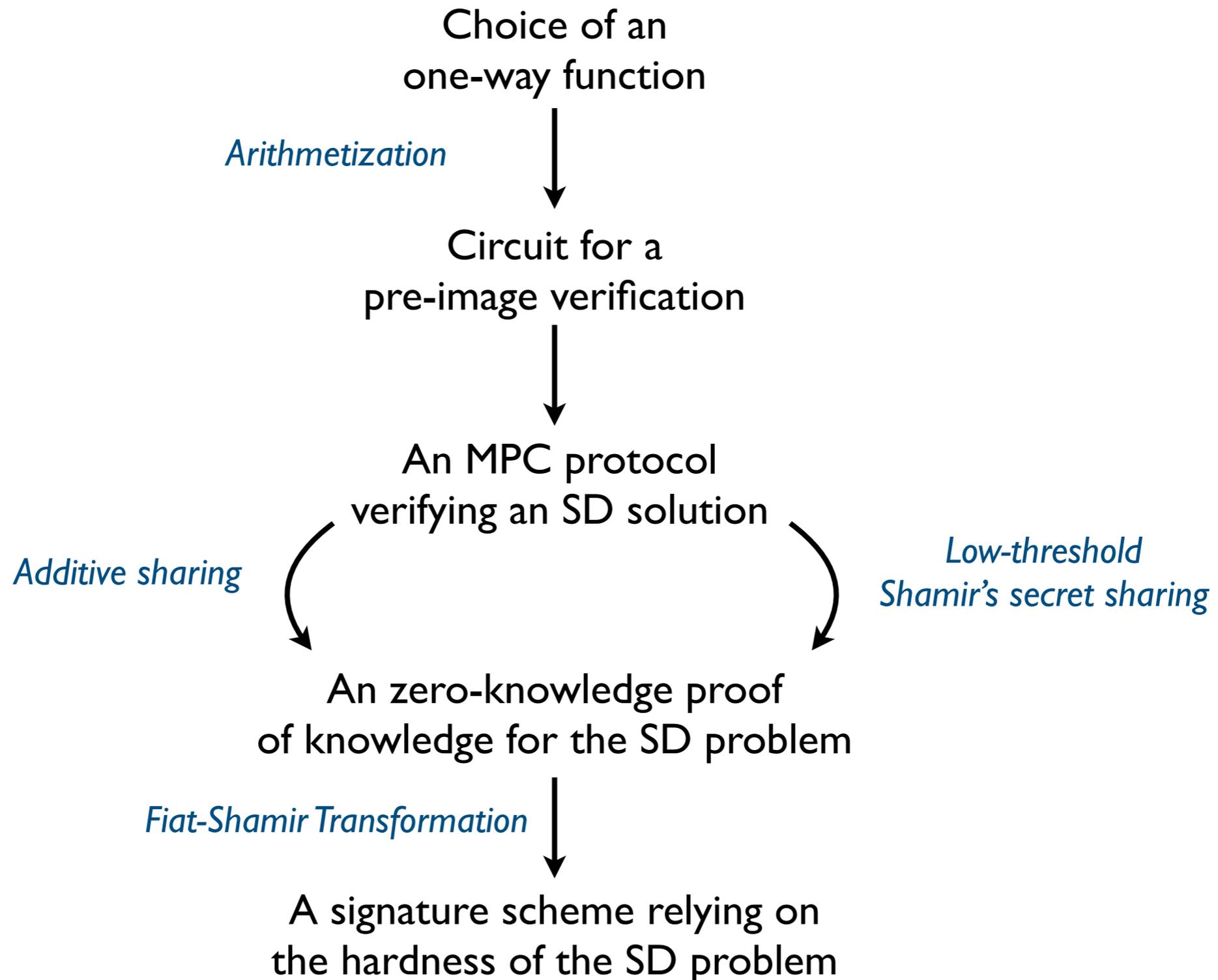
Fiat-Shamir Transformation



A signature scheme relying on the hardness of the SD problem



Build a signature scheme from MPC



An MPC-friendly statement

- Size of the solution ring: $\text{size}_{bits} \geq \lambda^2 + \frac{\log_2 |\text{ring}|}{\log_2 N} \cdot \lambda$
 - Lattice (SIS): ring of $2^{65\,536}$ solution candidates $\Rightarrow \text{size}_{bits} \geq 133 \text{ KB}$
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- Number of multiplications in the verification circuit

Signature scheme: SD-in-the-Head

- Many (standard) MPCitH optimisations to reduce the signature size
- Obtained signature sizes:

Field	PK size	Signature Size	
		Additive	LSSSitH
GF(2)	90-100 B	11-13 KB	-
GF(251)	140-150 B	8-9 KB	9.5-10.5 KB
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Why $GF(251)$ or $GF(256)$?

- Additive: the computational bottleneck is the pseudo-random generation (and the commitments). $GF(256)$ will be more efficient than $GF(251)$
- LSSSiTH: the computational bottleneck is the arithmetics. $GF(251)$ will be more efficient than $GF(256)$, especially on platforms without GFNI.

Performances

NIST Candidate SD-in-the-Head: Benchmark on a 2.60GHz recent platform

	Additive Sharing			LSSSiTH		
	Size	Sign	Verify	Size	Sign	Verify
SDitH (256)	8 241	5.18	4.81	10 117	1.97	0.62
SDitH (251)	8 241	8.51	8.16	10 117	1.71	0.23

Size in bytes, timing in milliseconds

Submitted candidates at NIST call

- Syndrome Decoding Problem:

SD-in-the-Head

C. Aguilar Melchor, T. Feneuil, N. Gama, S. Gueron, J. Howe, D. Joseph,
A. Joux, E. Persichetti, T. Randrianarisoa, M. Rivain, D. Yue.

- Rank Syndrome Decoding Problem:

RYDE

N. Aragon, M. Bardet, L. Bidoux, J.-J. Chi-Domínguez, V. Dyseryn,
T. Feneuil, P. Gaborit, A. Joux, M. Rivain, J.-P. Tillich, A. Vinçotte.

- Min Rank Problem:

MIRA

N. Aragon, M. Bardet, L. Bidoux, J.-J. Chi-Domínguez, V. Dyseryn,
T. Feneuil, P. Gaborit, R. Neveu, M. Rivain, J.-P. Tillich.

- Multivariate Quadratic Problem:

MQOM: MQ on my Mind

T. Feneuil, M. Rivain

Submitted candidates at NIST call

- How to deal with the rank metric [Fen22]:

- Technique 1: let us have a matrix $X \in \mathbb{F}_q^{n \times m}$

$$\text{rk}(X) \leq r \iff \exists T \in \mathbb{F}^{n \times r}, R \in \mathbb{F}^{r \times m} : X = T \cdot R$$

- Technique 2: let us have a vector $x \in \mathbb{F}_{q^m}^n$

$$w_R(x) \leq r \iff \exists P(X) := X^{q^r} + \sum_{j=0}^{r-1} \beta_j X^{q^j} : \forall i : P(x_i) = 0$$

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Shorter size



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Lighter scheme

Shorter size

- Rank SD: Technique 2 is the best

- From $H \in \mathbb{F}_{q^m}^{(n-k) \times n}$ and $y \in \mathbb{F}_q^{n-k}$, find a vector $x \in \mathbb{F}_{q^m}^n$ such that $y = Hx$ and $w_R(x) \leq r$.

- Min Rank: not clear which technique is the best

- From $M_0, M_1, \dots, M_k \in \mathbb{F}_q^{m \times n}$,

find a vector $x \in \mathbb{F}_q^k$ such that $\text{rk}(M_0 + \sum_{i=1}^k x_i M_i) \leq r$.

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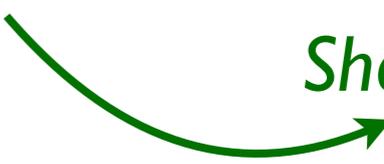
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Technique 1: MiRith
Technique 2: MIRA

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MQOM (251)	6 575	10.97	10.50	~ 14 000	-	-
RYDE	5 956	8.58	7.31	~ 9 200	-	-
MIRA (16)	5 640	16.65	15.61	-	-	-

*Size in bytes, timing in milliseconds
Isochronous implementations
Single thread*

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- Dilithium: $|\text{sig}|=2420$, $|\text{pk}|=1312$, $t_{\text{sign}}=0.13$, $t_{\text{verify}}=0.05$
- Falcon: $|\text{sig}|=666$, $|\text{pk}|=897$, $t_{\text{sign}}=0.20$, $t_{\text{verify}}=0.06$
- SPHINCS+: $|\text{sig}|=7856$, $|\text{pk}|=32$, $t_{\text{sign}}=331$, $t_{\text{verify}}=2.3$
 $|\text{sig}|=17088$, $|\text{pk}|=32$, $t_{\text{sign}}=19$, $t_{\text{verify}}=0.9$

Conclusion

■ MPC-in-the-Head

- A practical tool to build *conservative* signature schemes
- Very versatile and tunable
- Can be applied on any one-way function

■ Perspectives

- *Additive-based MPCitH*: stable
- *Low-threshold-based MPCitH*: new approach, could lead to follow-up works